Real Balance Effects When the Nominal Interest Rate is Zero.

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Abstract
What are the economic effects of a large increase in real balances of money on the economy when the nominal interest is zero? This paper considers this question using a computable overlapping generations model. In this model increases in money supply that increase the overall stock of government debt have real effects. Results from a dynamic simulation analysis indicate that monetary policies such as quantitative easing that temporarily increase real balances of money act to raise the real interest rate and thus crowd out private capital formation. This can depress economic activity and increase deflationary pressure but the magnitude of these responses is small. Quantitative easing can have pronounced distributional effects. It benefits the old, but hurts middle-age workers.

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1 Introduction

This paper uses an economic model to assess the effects of a large empirically relevant increase in real balances of narrow money on the price level, economic activity and welfare when the nominal interest rate is zero. We will refer to such a monetary policy as “quantitative easing.” This topic is motivated by recent events in Japan and the United States. In Japan after the nominal interest rate fell to zero in 1999, the Bank of Japan adopted a quantitative easing monetary policy that saw the ratio of M0 to GNP rise from 0.13 to 0.22 over the next six years. In the United States the ratio of M0 to output has nearly doubled rising from 0.08 to 0.14 since the Federal Funds rate fell to effectively zero in the fourth quarter 2008.

The experiences of Japan and the U.S. have pointed to a rather glaring omission in some of the most widely-used empirical models of monetary economics. Following Woodford (2003), it has become common to abstract from money entirely and consider questions of monetary policy in a cashless economy. Even if money is introduced into the model via money in the utility function or a cash in advance constraint, money doesn’t matter once the nominal interest rate is zero. Money and bonds become perfect substitutes and a change in the timing of total government liabilities (including money) and lump-sum taxes has no effects on prices or economic activity in infinite horizon representative agent models.

Ireland (2005) suggests one way to introduce real balance effects. He conducts a steady-state analysis in a Blanchard (1985) model with overlapping generations of infinite-lived individuals. In that model positive population growth breaks Ricardian equivalence and the growth rate of money matters even when the nominal interest rate is zero.

We consider real balance effects in an overlapping generations (OG) model with a cash in advance constraint. Two distinct factors produce real balance effects in our model when the nominal interest rate is zero. First, individuals

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1 Quantitative easing was the term used by the Bank of Japan to describe the monetary policy it pursued from March 19, 2001 thru March 2006.

2 Auerbach and Obstfeld (2005) provide another reason for why quantitative easing might matter that is not pursued here. When the nominal interest rate is zero a surprise open-market purchase of bonds can counteract deflationary price tendencies and lower the real value of government debt if households expect that the nominal interest rate will eventually rise above zero. Lower government debt reduces the need to rely on other distortionary taxes and this can raise household welfare.
have finite lifetimes and the timing of government borrowing can affect their present value tax liabilities and induce wealth effects. A second channel arises from the presence of borrowing constraints. When an individual faces a binding borrowing constraint, policies that lower current taxes can increase disposable income and current consumption.

We use computational methods to solve for the competitive equilibrium. This approach makes it possible to entertain empirically relevant model time periods and compare model prices and allocations directly with actual data. We assume the model period is a year, and that individuals are active for 80 years.

Our dynamic simulation results suggest that quantitative easing monetary policies act to increase deflationary pressure, lower economic activity and increase the number of years that the nominal interest rate is zero.

To understand the reason for this finding, consider first the steady-state properties of our model. In our model there are multiple growth rates of money that are consistent with a zero nominal interest rate and each of these steady-states delivers a different set of prices and allocations. Within this set, the steady-state with the highest growth rate of money that is consistent with a zero nominal interest rate maximizes lifetime welfare. We refer to this monetary policy as the Friedman rule.

The welfare costs of deviations from this policy are asymmetric. On the one hand, high money growth induces an inflation tax on households who are required to effect a fraction of their purchases with cash. In our model the welfare costs of high money growth are modest.

On the other hand, if the growth rate of money is lowered below its optimal value welfare falls sharply. Once the nominal interest rate reaches zero, money competes directly with capital for individuals’ savings. In this situation a lower growth rate of money increases the real return on money and crowds out private capital. We refer to this as a fiscal effect.

The steady-state analysis offers valuable insights into the mechanisms at work but misses some important factors. For plausibly parameterized versions of the model zero nominal interest rates go hand in hand with deflation and negative growth of the money supply. In both Japan and the United States, in contrast, quantitative easing has been associated with big increases in nominal money supply. Moreover, the quantitative easing policies pursued by both Japan and the United States are explicitly temporary.

We thus turn to conduct a dynamic perfect foresight analysis using Japanese data. Japan is an interesting reference point because it has been experiencing
deflation since the late 1990s. During much of this period (1999-2006) the nominal interest rate was zero and yet real balances of money grew sharply.

After calibrating the model to Japanese data we display a baseline dynamic simulation that successfully reproduces some of the principal economic developments in the Japanese economy between 1990 and 2006. The model reproduces the slow down in real activity during this period. It also reproduces nominal developments including deflation and the protracted period of zero nominal interest rates against a background of a large increase in the ratio of real balances to output.

Having produced an empirically relevant empirical specification of our model we next use the model to analyze the effects of quantitative easing using a counterfactual analysis. This analysis indicates that the fiscal effect identified in the steady-state analysis also plays an important role in a dynamic setting.

Starting from an initial situation with zero current and expected future nominal interest rates, a counterfactual with additional quantitative easing (higher real balances) has a dominant fiscal effect. Higher real balances are associated with a higher growth rate of money and this increases the real value of government debt. In order to get households to hold more of their savings in the form of money the real interest rate has to rise. This crowds out private capital. A higher return on capital lowers the wage rate and this acts to depress economic activity. A higher return on money also produces more deflation and a longer period of zero nominal interest rates.

Quantitative easing has important effects on individuals that vary with age. A higher real interest rate benefits retirees. Their savings now have a higher return. They also experience a second benefit. Higher debt means temporarily lower taxes. The old face high mortality rates, and thus largely escape higher future taxes.

A higher real interest rate is also associated with a lower wage rate and this acts to lower consumption for working individuals. The negative effects of a lower wage turn out to be most pronounced for middle-aged workers who are close to their peak lifetime labor efficiency.

Temporarily high government debt and high interest rates can ease borrowing constraints. In our model the young are borrowing constrained. They face an increasing wage profile but cannot collateralize their future human capital. Higher debt implies that taxes are lower today and this allows young agents to consume more. A higher real interest rate induces an intertemporal substitution effect that can also benefit the young by reducing demand for
today’s consumption. These two positive effects for the young, though, get balanced against the negative effect of a lower wage.

We find that the biggest winners and losers are concentrated among older individuals. The higher real interest rates associated with quantitative easing increase consumption of retirees by as much as 2.3 percent between 2001 and 2005. Workers who have the highest labor productivity, though, face lower wages and experience annual consumption losses of as much as 1.3 percent. For younger workers the benefits of relaxed borrowing constraints are largely offset by lower wages and the consumption gains are small.

The remainder of the paper is organized as follows: Section 2 describes the model; Section 3 explains how we parameterize the model; Section 4 contains the steady-state results; and Section 5 the dynamic results. Our concluding remarks are in Section 6.

2 The Model

We consider an economy that evolves in discrete time. The structure of the real side of the economy is similar to the economy considered by Braun, Ikeda and Joines (2009). Their model accounts for many of the principal variations in macroeconomic activity in the Japanese economy between 1961 and 2002.

2.1 Demographics

Agents are born and become active at age 21. The growth rate of 21 year old individuals, $n_1$ is assumed to be constant in each period. Agents are subject to mortality risk in each period. If we let $N_{j,t}$ be the number of households of age $j$ in period $t$, the dynamics of population are governed by a first-order Markov process:

$$
N_{t+1} = \begin{bmatrix}
(1 + n_1) & 0 & 0 & \ldots & 0 \\
\psi_1 & 0 & 0 & \ldots & 0 \\
0 & \psi_2 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \psi_{J-1} & 0
\end{bmatrix} N_t \equiv \Gamma N_t
$$

(1)

where $N_t$ is a $J \times 1$ vector that describes the population of each cohort in period $t$, $\psi_j$ is the conditional probability that a household of age $j$ survives.
to the next period and \( \psi_j \) is implicitly assumed to be zero. The aggregate population in period \( t \), denoted by \( N_t \), is given by

\[
N_t = \sum_{j=1}^{J} N_{j,t}
\]

(2)

The population growth rate is then given by \( n = N_{t+1}/N_t \). The unconditional probability of surviving from birth in period \( t - j + 1 \) to age \( j > 1 \) in period \( t \) is:

\[
\xi_j = \psi_{j-1} \xi_{j-1}
\]

(3)

In words, survival probabilities depend on age but not on the year of birth.

### 2.2 Individual’s Problem

Individuals are born at age 21 with zero assets and retire at age 65. The maximum life-span of an individual is \( J = 100 \) years. Money is introduced by assuming that households receive utility from two goods. The cash good, \( c_{1t} \), is subject to a cash in advance constraint as in Lucas and Stokey (1987). The credit good, \( c_{2t} \), may be purchased with cash or on credit. Households also value leisure, \( l_{jt} \). Given these definitions expected present value utility of a household belonging to cohort \( s \) is:

\[
U_s = \sum_{j=1}^{J} \beta^{j-1} \xi_j u(c_{1jt}, c_{2jt}, l_{jt})
\]

(4)

We assume the period utility function is given by:

\[
u(c_{1jt}, c_{2jt}, l_{jt}) = \gamma \ln(c_{1jt}) + (1 - \gamma) \ln(c_{2jt}) + \alpha \ln(l_{jt}) \]

(5)

This choice of preferences is consistent with balanced growth.\(^3\) An individual, of age \( j \) in period \( t \), who works \( h_t \) hours receives nominal earnings of \( P_t w_t \varepsilon_j h_{jt} \). In this expression \( P_t \) is the price level, \( w_t \) is the wage rate, and \( \varepsilon_j \) is an age specific efficiency. A household can save by accumulating cash, \( M_{t+1} \), bonds \( B_{t+1} \), or capital \( k_{t+1} \).

\(^3\)More generally preferences of the form: \( \ln([\gamma(c_{1jt})^\sigma + (1 - \gamma)(c_{1jt})^\sigma]^{1/\sigma}) + \alpha \ln(1 - n_t) \) are also consistent with balanced growth.
At the start of each period households visit a financial market where claims from the previous period are settled. Households also receive a lump-sum transfer from the government \( T_{jt} \), and adjust their holdings of money and bonds. Total holdings of assets are restricted by the following borrowing constraint:

\[
P_t k_{j,t+1} + B_{j,t+1} + M_{j,t+1} \geq 0. \tag{6}
\]

This borrowing constraint rules out uncollateralized borrowing.

After the financial market closes households separate into a worker and shopper. The shopper’s purchases of the cash good and investment goods in any period are subject to the following cash in advance constraint:

\[
\frac{B_{j,t+1}}{1 + R_t} + P_t [k_{j,t+1} - k_{j-1,t}] + P_t c_{1jt} \leq M_{j-1,t} + T_{jt} + B_{j-1,t} + P_t (1 - \tau)(r_t - \delta)k_{j-1,t} \tag{7}
\]

where \( \delta \) is the depreciation rate on capital and \( \tau \) is a tax on capital income.\( ^4 \)

The individual’s overall budget constraint is given by:

\[
M_{j-1,t} + T_{jt} + B_{j-1,t} + P_t w_t \varepsilon_j h_{jt} + P_t (1 - \tau)(r_t - \delta)k_{j-1,t} \geq P_t (c_{1jt} + c_{2jt}) + \frac{B_{j,t+1}}{1 + R_t} + M_{j,t+1} + P_t [k_{j,t+1} - k_{j-1,t}] \tag{8}
\]

Given these definitions the problem for an individual born into cohort \( s \) is to choose the sequence \( \{c_{1t}, c_{2t}, h_t, M_{t+1}, B_{t+1}, k_{t+1}\}_{t=s} \) that maximizes subject to (6), (7), and (8). Some key household first order necessary conditions are:

\[
\xi_j \gamma / c_{1jt} = P_t (\mu_{j,t} + \lambda_{j,t}) \tag{9}
\]

\[
\xi_j (1 - \gamma) / c_{2jt} = P_t \lambda_{j,t} \tag{10}
\]

\[
\xi_j \frac{\alpha}{1 - h_{jt}} = \lambda_{j,t} P_t w_t \varepsilon_j \tag{11}
\]

\[
\beta (\mu_{t+1} + \lambda_{t+1}) + \phi_t = (\mu_{j,t} + \lambda_{j,t}) / (1 + R_t) \tag{12}
\]

\[
\beta P_{t+1} (\lambda_{j,t+1} + \mu_{j,t+1}) |1 + (1 - \tau)(r_{t+1} - \delta)| + \phi_{j,t} = (\lambda_{j,t} + \mu_{j,t}) P_t \tag{13}
\]

\[
\beta (\mu_{j,t+1} + \lambda_{j,t+1}) / + \phi_{j,t} = \lambda_{j,t} \tag{14}
\]

\( ^4 \) Including investment in the cash in advance constraint will act to exaggerate the welfare costs of inflation. We will show below, however, that these welfare costs are quantitatively small as compared to the welfare costs of too much deflation.
plus the CIA constraint, household budget constraint and the borrowing constraint. In the above equations $\phi_{j,t}$, $\mu_{j,t}$ and $\lambda_{j,t}$ denote the La Grange multipliers on respectively equations (6)-(8).

Let $\pi_t \equiv p_t / p_{t-1}$. Then the above expressions can be rearranged to yield the following restrictions on market clearing:

$$\frac{\alpha}{\gamma} \frac{c_{1jt}}{1 - h_{jlt}} = \frac{w_t \varepsilon_j}{(1 + R_t)}$$  \hspace{1cm} (15)

$$\frac{\alpha}{1 - \gamma} \frac{c_{2jt}}{1 - h_{jlt}} = \frac{w_t \varepsilon_j}{(1 + R_t)}$$  \hspace{1cm} (16)

$$\beta [1 + (1 - \tau_{t+1})(r_{t+1} - \delta)] \xi_{j+1} / c_{1j,t+1} = \xi_j / c_{1j,t} - \phi_t$$  \hspace{1cm} (17)

$$(1 + R_t) / (1 + \pi_{t+1}) = 1 + (1 - \tau)(r_{t+1} - \delta)$$  \hspace{1cm} (18)

$$\phi_t (M_{j,t+1} + P_t k_{j,t+1} + B_{j,t+1}) = 0, \quad \phi_t \geq 0$$  \hspace{1cm} (19)

$$\mu_{j,t} \left\{ \begin{array}{l} B_{j-1,t} - \frac{B_{j,t+1}}{1 + R_t} + (1 - \tau)(r_t - \delta) k_{j-1,t}^t + \\ P_t [k_{j-1,t}^t - k_{j,t+1}^t] + M_{j-1,t} + T_{j,t} - P_t c_{1jt}^t \end{array} \right\} = 0, \quad \mu_t \geq 0$$  \hspace{1cm} (20)

$$M_{j-1,t} + T_{j,t} + B_{j-1,t} + P_t w_t \varepsilon_j h_{j,t} + P_t (1 - \tau)(r_t - \delta) k_{j-1,t}^t =$$

$$P_t (c_{1t} + c_{2t}) + \frac{B_{j,t+1}}{1 + R_t} + M_{j,t+1} + P_t [k_{j,t+1} - k_{j-1,t}^t]$$  \hspace{1cm} (21)

### 2.3 The Firm’s Problem

Firms produce consumption goods with a constant returns to scale production technology. In each period firms choose labor, $H_t$, and capital, $K_t$, to maximize

$$A_t K_t^\theta H_t^{1-\theta} - w_t H_t - r_t K_t,$$  \hspace{1cm} (22)

where $w_t$ is the real wage and $r_t$ is the real rental rate on capital, $A_t$ evolves according to

$$A_{t+1} = g_t A_t, g_t > 0.$$
2.4 The Government and aggregate feasibility constraints

The government issues bonds, money and raises revenue through a tax on asset income. Government revenue is used to finance government purchases and lump-sum transfers:

\[
P_t G_t + \sum_{j=1}^{J} N_{jt} T_{jt} = \frac{B_{t+1}}{1+R_t} - B_t + M_{t+1} - M_t + P_t \tau (r_t - \delta) K_t
\]

(23)

The government expands (nominal) money supply at the rate \(\sigma_t\) by making lump-sum transfers to all households alive in a given period according to:

\[M_{t+1} = (1 + \sigma_t) M_t.\]

We don’t formally model a social security system. Instead we will assume that accidental bequests are lump-sum transferred back to surviving members of the same cohort.

The aggregate resource constraint for this economy is:

\[
A_t K_t^\theta H_t^{1-\theta} = \sum_{j=1}^{J} N_{jt} (c_{1jt} + c_{2jt}) + K_{t+1} - (1 - \delta) K_t + G_t
\]

(24)

2.5 Competitive Equilibrium

**Definition** Competitive Equilibrium

Given an initial population wealth distribution, \(\{M_{0j}, k_{0j}, B_{0j}\}_{j=1}^{J}\), a sequence of technologies, \(\{A_t\}_{t=0}^{\infty}\), and government policies, \(\{\tau, M_{t+1}, B_{t+1}, G_t, \{T_{jt}\}_{j=1}^{J}\}_{t=0}^{\infty}\), a competitive equilibrium is a price system \(\{r_t, P_t, R_t, w_t\}_{t=0}^{\infty}\) and a sequence of allocations \(\{c_{jt}, h_{jt}, k_{j,t+1} + b_{j,t+1}, M_{j,t+1}\}_{t=0}^{\infty}\) that solves the household and the firm’s problem, and satisfies the following market clearing/feasibility conditions:
\[ B_{t+1} + K_{t+1} = \sum_{j=1}^{J} N_{j,t} (k_{j,t+1} + b_{j,t+1}) \]  

(25)

\[ H_t = \sum_{j=1}^{J} N_{j,t} h_{jt} \]  

(26)

\[ M_{t+1} = \sum_{j=1}^{J} N_{j,t} M_{j,t+1} \]  

(27)

\[ A_t K_t^\theta H_t^{1-\theta} = \sum_{j=1}^{J} N_{j,t} (c_{1jt} + c_{2jt}) + K_{t+1} - (1 - \delta_t)K_t + G_t \]  

(28)

When solving the model we will specify an initial population wealth distribution and a terminal steady-state and then solve for the transitional dynamics. We thus define a steady-state equilibrium next.

**Definition Balanced Growth Equilibrium**

Suppose that technology grows at the constant rate: \( g_t = g \), and that money supply grows at a constant rate: \( \sigma_t = \sigma \), and the output shares of government purchases, and government debt are constant. Then a balanced growth equilibrium is a competitive equilibrium in which the real wage rate grows at the rate of output, the real interest and nominal interest rates are constant and the output shares of capital and consumption are constant.

### 2.6 Computation of the equilibrium.

Before we compute the equilibrium we transform the economy. This is done using the transformations:

\[ \hat{K}_t = \frac{K_t}{N_t A_t^{1/(1-\theta)}}; \hat{C}_t = \frac{C_t}{N_t A_t^{1/(1-\theta)}}; \hat{B}_t = \frac{B_t}{P_{t-1} N_t A_t^{1/(1-\theta)}}; \]

\[ \hat{M}_t = \frac{M_t}{P_{t-1} N_t A_t^{1/(1-\theta)}}; \hat{T}_t = T_t / P_t; \hat{H}_t = H_t / N_t; \hat{w}_t = \frac{w_t}{A_t^{1/(1-\theta)}}. \]  

(29)

We first describe computation of the steady-state equilibrium. We are interested in considering situations where the nominal interest rate is positive and also in situations where it is zero. In the latter situation the cash in advance
constraint (7) ceases to bind and the steady-state conditions are different. When \( R > 0 \) we start by guessing the aggregate values of hours, capital, real balances and lump-sum transfers \((\hat{H}_0, \hat{K}_0, \hat{M}_0, \hat{T}_0)\). Given these objects we can derive the wage and rental rates \( \bar{w}_0, r_0 \) and solve the household’s problem. Note that the inflation rate can be derived from real balances using the following equation:

\[
(1 + \pi) = \frac{(1 + \sigma)}{(1 + n)(1 + g_{TFP})}
\]  

(30)

where \( 1 + g_{TFP} = A_t^{1/(1-\theta)}/A_{t-1}^{1/(1-\theta)} \). When \( R > 0 \), the solution to the household’s problem uniquely determines individual demand for real balances \( \hat{M}_0^{d,s} \) for each cohort \( s = \{1, \ldots, J\} \) and labor supply for each cohort \( \hat{H} \). However, the household’s problem only determines the sum of saving in the form of capital and bonds. We denote this sum as \( \hat{S}_0 \).

Given solutions to each cohort’s optimization problem we then sum over households to derive aggregate assets supplied by households: \( \hat{S}'_0 \), aggregate labor supply: \( \hat{H}'_0 \) and aggregate demand for real balances: \( \hat{M}'_0 \). Given these objects we can solve for the capital stock using the fact that the stock of government bonds is exogenous and:

\[
\frac{(1 + g_{TFP})(1 + n)}{1 + R} - \frac{1}{1 + \pi} \hat{B} + \frac{(1 + g_{TFP})(1 + n) - \frac{1}{1 + \pi}}{1 + \pi} \hat{M}' + \tau (r - \delta) \hat{K}' = \hat{G} + \hat{T}'.
\]  

(31)

Finally, we update our guess of capital, labor, real balances and government transfers by taking a weighted average of the initial guess plus the new values derived from household optimization:

\[
\hat{K}_1 = \lambda \hat{K}'_0 + (1 - \lambda) \hat{K}_0
\]  

(32)

\[
\hat{H}_1 = \lambda \hat{H}'_0 + (1 - \lambda) \hat{H}_0
\]  

(33)

\[
\hat{T}_1 = \hat{T}'_0
\]  

(34)

\[
\hat{M}_1 = \lambda \hat{M}'_0 + (1 - \lambda) \hat{M}_0
\]  

(35)

When \( R = 0 \), the household problem only pins down household supply of total assets which now consists of the sum of real balances, capital and
bonds: \( \hat{S}_0' = \hat{K}_0' + \hat{B} + \hat{M}_0' \). In this case we derive aggregate real balances and capital in the following way. First, we use the fact that:

\[
(1 + \pi_0) = \frac{(1 + \sigma)}{(1 + n)(1 + g_{TFP})} \tag{36}
\]
to pin down the inflation rate. Then we use

\[
(1 + \pi_0) = (1 + r_0)^{-1} \tag{37}
\]
to pin down the real interest rate. Given the real interest rate we derive a new guess of the capital stock, \( \hat{K}_0' \), from aggregate labor supply plus the marginal product pricing relationship:

\[
r_0 = (1 - \tau) \left\{ \theta \left( \frac{\hat{K}_0'/\hat{H}_0'}{\hat{K}_0'/\hat{H}_0'} \right)^{a-1} - \delta \right\} \tag{38}
\]

Then we derive real balances from the saving identity: \( \hat{S}_0' - \hat{K}_0' - \hat{B} = \hat{M}_0' \). The updating of the guess proceeds in the same way as before.

When solving for the dynamic transition we proceed in an analogous way. The main distinction is that we now guess and update sequences of the form: \((\hat{H}_{i,t}, \hat{K}_{i,t}, \hat{M}_{i,t}, \hat{T}_{i,t})\) where \( i \) denotes the \( i^{th} \) iterate and \( t \) indexes time.

### 3 Model Parameterization

The strategy for calibrating the model is similar to the strategy used in Braun, Joines and Ikeda (2009). The preference discount rate \( \beta \) is calibrated to reproduce the average capital output ratio between 1984 and 2000. This results in a value of 0.97. The leisure weight in preferences, \( \alpha \) is set to reproduce the value of labor input in the Japanese economy between 1984 and 2000. This yields \( \alpha = 2.5 \). The capital share parameter is set to 0.362 which is the average value of capital’s share of GNP between 1984 and 2000. The depreciation rate, which is calibrated in the same way, is 0.085. The average tax rate on asset income over the same period is 0.46. The labor tax rate is set to zero. This assumption is also maintained by Hayashi and Prescott (2002) who assert that the principal tax wedge in Japan is a high tax on capital income. With this choice, the remainder of the calibration turns out to be very similar to what one finds using U.S. data. We assume a constant population growth rate of 1 percent per year. We set the share
weight on cash goods, $\gamma = 0.07$. This choice reproduces the ratio of real balances of monetary base to GNP which averaged 0.08 between 1984 and 1994.

Details about the setting of the sequences of exogenous variables and the initial wealth-population distribution that are specific to the dynamic analysis are reported below in Section 5.

4 Steady-state Analysis

Here we report results from a comparative steady-state analysis. These results provide some useful intuition about the workings of the model that turns out to be very helpful for understanding the dynamic results.

Table 1 reports the steady-state properties of our model for alternative settings of the growth rate of money. These results allow for age specific variation in the efficiency of work effort and assume that the population growth rate is 1 percent, the growth rate of TFP is 1.9 percent, the share of government purchases in output is 0.144 and the government debt ratio is 0.22. These correspond to the average values of these variables in Japanese data over the 1984 to 2000 sample period.

Observe that the welfare maximizing choice of the growth rate of money is $-1.43$ percent per year. The associated value of the nominal interest rate is zero. In this sense the Friedman rule is the optimal monetary policy for this economy.

Bhattacharya, Haslag and Russell (2005) consider the optimality of the Friedman rule in a 2 period overlapping generations model and find that it is not optimal in their setting. The reason for this is that in their model young households have low initial wealth and yet must pay a lump-sum tax to finance contraction of the money supply when the growth rate of money is negative. Ireland (2005) reaches a similar conclusion in a setting with overlapping generations of infinitely lived agents. We allow agents to borrow against their first period labor earnings and this acts to mitigate the negative effect of lump-sum taxation on welfare of the youngest households.

In standard infinite horizon models there is only one choice of the growth rate of money that yields a zero nominal interest rate. In this economy, however, there is a range of values of the growth rate of money that is consistent with a zero nominal interest rate. Interestingly, welfare falls rapidly if the growth rate of money is too low.
To understand why welfare falls when the growth rate of money is too low, recall that when the nominal interest rate is zero the cash in advance constraint ceases to bind and money and capital earn the same real return. As the steady-state growth rate of money is lowered from its welfare maximizing level, the inflation rate falls and this increases the real return on money. Holdings of private capital must then fall in order to insure that capital continues to earn the same return as money. Since these responses are due to an increase in the overall stock of government indebtedness, we will refer to this as a fiscal effect.

Inspection of the final column of Table 1 indicates that the welfare cost of too much deflation is quite substantial. The welfare cost of a growth rate of money of \(-1.8\) or 0.4 percent below the optimal growth rate of money is 1.82 percent of expected lifetime consumption.\(^5\)

Life-time consumption profiles shift in interesting ways. The policies when young don’t change much. Instead the most pronounced effects occur on policies for ages between 40 and 50. Consumption and leisure both fall by as much as 3 percent when comparing the optimal policy with the \(-1.8\) percent money growth scenario. An important reason for these responses is that a higher real return lowers the wage rate. This has the most pronounced effect on decisions in the periods of life where labor efficiency is high.

The welfare cost of too rapid growth on money supply is relatively small. When the average growth rate of money is higher than the optimal level, monetary policy acts as a tax on labor supply and capital. Following the previous literature we refer to this effect as an inflation tax effect. Households act to limit their holdings of cash and this limits the incidence of this tax. These effects can readily be observed in Table 1. Higher growth rates of money are associated with lower consumption of cash goods. However, cash goods only constitute 7 percent of total consumption under the Friedman rule so this effect is modest. The effect of too much inflation on the capital output ratio and the real interest rate is also modest. The result is that the welfare cost of too much inflation is relatively small. For instance, the welfare cost of 7 percent money growth (4 percent inflation) is 0.89 percent of lifetime consumption. This is only marginally higher than the welfare cost of \(-1.6\) percent deflation.

We wish to emphasize that the estimated welfare costs of too much steady

\(^5\)We calculate consumption equivalent welfare costs relative to the optimal steady-state policy. The consumption equivalents are total consumption equivalents.
inflation reported in Table 1 are, if anything, on the high side. This is because investment has been treated as a cash good. The welfare costs of inflation would be even lower if investment was treated as a credit good instead.

This asymmetry has implications for the conduct of monetary policy. Suppose we assume that the monetary authority knows the model but that there is uncertainty about the values of the model parameters including the long run average values or growth rates of the exogenous variables. To be specific suppose that the policy maker estimates the growth rate of TFP is 4 percent rather than 2 percent. An estimate of this magnitude would arise if the policy maker were to estimate the growth rate of TFP using Japanese data from 1960 to 1990. In this scenario welfare is maximized when the growth rate of money is $-0.61$ percent and also falls rapidly if the money supply is contracted more rapidly. Setting the growth rate of money to $-1.43$ percent, which is the optimal monetary policy in Table 1, induces very large welfare losses. This property of the model provides a rationale for a monetary authority to pursue an expansionary monetary policy when it finds itself in a zero interest rate environment. The welfare costs associated with too much monetary expansion are much smaller than the welfare costs of a monetary policy that is too tight.

The steady-state results are very helpful for building intuition about the properties of the model. This mode of analysis has some important limitations. For instance, it is difficult to produce an empirically plausible calibrated specification of the model with a steady-state in which a zero nominal interest rate is associated with deflation and a positive growth rate of money. Thus it is difficult to use a comparative steady-state analysis to understand Japan’s experience from the mid 1990s to 2006 when the growth rate of money was positive and yet there was a protracted period of deflation. In addition, the quantitative easing monetary policies pursued in Japan and the United States are explicitly temporary. For these reasons we now turn to describe the results from a dynamic analysis.

5 A Dynamic Analysis of ”Quantitative Easing”

Japan is an interesting case for analyzing the empirical effects of a large increase in real balances of money in a zero interest rate environment. In
Japan slower real economic growth during the 1990s was associated with a steady decline in the uncollateralized call rate on overnight loans from 7.4 percent in 1990 to 0.06 percent in 1999. The nominal interest rate remained at effectively zero (except for a brief interlude in 2000) until 2006. Once the nominal interest rate reached zero policy makers considered a variety of options for using monetary policy to stimulate the economy. The outcome of these deliberations was the “Quantitative Easing” policy that was adopted on March 19, 2001. This policy which targeted the level of bank deposits at the Bank of Japan was effectively an excess reserve targeting policy. The Bank of Japan announced an end to the quantitative easing policy in March 9, 2006. But it kept the call rate at zero until July 14, 2006 at which point the call rate was increased to 0.25 percent.

We investigate the effects of this policy using dynamic perfect foresight simulations. Chen, Imrohoroglu and Imrohoroglu (2007) and Braun, Ikeda and Joines (2009) have previously found that computable general equilibrium models that allow for variation in TFP and demographics can account for some of the principal movements in real economic activity in Japan from 1960 through 2002. Here we abstract from demographic variation and model variation in TFP, government purchases and government debt. Our government debt series is taken from Braun, Joines and Ikeda (2009). They construct a net government debt series using the methodology of Broda and Weinstein (2005). The resulting government debt output ratio is 0.29 in 1997 and rises to 0.85 in 2005. The initial period of our simulation is taken to be 1984. The initial wealth distribution is taken from the terminal steady-state but is rescaled to reproduce the capital stock in Japanese data in 1984. We set the initial values of the nominal interest rate, government purchases and government bonds to their values in Japanese data in 1984. The terminal nominal interest rate is 5.9 percent, terminal government debt is 22 percent of output, terminal government purchases are 14.4 percent of output and terminal TFP growth is 1.9 percent. We assume that each of these variables returns to its steady-state value according to a linear rule between 2007 and 2015.

Chen, Imrohoroglu and Imrohoroglu (2007) conduct a comparison of the perfect foresight assumption we maintain here with some alternative ways of forming expectations and find that the results are similar. In our case, computational considerations make it difficult for us to entertain alternative assumptions. In some of the simulations one run can take as long as two weeks. This cost is due to the fact that both the number of years any given cohort is borrowing constrained is endogenous as well as the number of periods that the nominal interest rate is zero.
We would like the model to reproduce variations in the Japanese Call Rate and real balances of money during the period 1986-2006.\footnote{Our measure of money is the monetary base. Personal checks don’t exist in Japan and about 80 percent of monetary base is currency in circulation.} There are two issues that arise in doing this. First, when the nominal interest rate is zero the composition of government liabilities is indeterminate. Open market operations that exchange money for bonds have no real effects when the nominal interest rate is zero in our perfect foresight setting. Monetary policies that alter the total amount of outstanding government debt do have real effects. However, it is hard to ascertain directly what fraction of quantitative easing should be interpreted as having altered the amount of outstanding government debt. During this period the Bank of Japan purchased equities of private companies, accepted a broader range of assets as collateral and purchased long-term bonds. Throughout most of this period the Bank of Japan was also the sole provider of funds in the overnight call money market. The reason for this was that the interest rate was so low that the return from lending overnight was dwarfed for all but very large loans by the costs of originating an overnight loan. These transactions costs killed one side of the private overnight interbank loan market. Effectively each overnight loan was supplied by the Bank of Japan at a subsidized rate.

Directly valuing all of these transactions is beyond the scope of this paper. Instead we use the model to infer the fraction of the total debt accumulation during this period that was associated with monetary policy. This is accomplished in the following way. For the period between 1984 and 1997 we treat the sequence of government debt as exogenous and choose the growth rate of money in the model to reproduce the nominal interest rate in Japanese data. From 1998 to 2006 the nominal interest rate is effectively zero so we solve the model conjecturing shares of total government debt that are attributed to monetary policy. We then update our guess of the share of total government liabilities until we reproduce the actual realization of the ratio of M0 to GNP in Japanese data between 1997 and 2006. The resulting model based calibration attributes a maximum of 21 percent of total government debt outstanding to the monetary authority in 2004.

The resulting trajectory for M0/P to GNP from the model and Japanese data are reported in Figure 1. This same figure also reports plots of the capital output ratio, the deviation of output from a 1.9 percent trend and the inflation rate as measured by the growth rate of the GNP price deflator.
The general fit of the model is reasonably good. The model reproduces the increase in the capital-output ratio and the decline in output relative to trend that Japan experienced after 1990. However, the model understates the average value of the inflation rate in Japanese data. There is a trade off in the model between matching the average level of real balances and the average inflation rate. Given that the focus of this paper is on real balance effects we chose to calibrate the model in a way that reproduces the average value of the former.

We evaluate the effects of quantitative easing by comparing the baseline simulation with three counterfactual simulations. The no quantitative easing scenario assumes that the ratio of real balances of M0 to GNP rises at the rate of 2 percent per year between 2000 and 2006. The longer quantitative easing scenario allows the ratio of real balances to output to rise to 0.24 in 2011. The earlier quantitative easing scenario assumes that quantitative easing starts in 1995 instead of 2002. Figure 2 shows the trajectory of inverse M0 velocity for each of these scenarios.

Table 2 summarizes properties of the model for prices. A comparison of the baseline simulation with the no quantitative easing simulation reveals two effects of quantitative easing. The quantitative easing simulation exhibits less deflation during the 1990s and a higher nominal interest rate than the no quantitative easing simulation. Expectations are clearly playing an important role. The baseline specification shows higher nominal interest rates and higher average inflation rates between 1991 and 2000 which is before quantitative easing was undertaken. After 2000 the two policies are very similar. Movements in the inflation rate are large and of primary importance in determining the evolution of the nominal interest rate. However, there are also some small but discernible effects on the real interest rate. The real interest rate is higher under quantitative easing in all sub-periods.

An interesting property of the model is that longer quantitative easing increases deflationary pressure. The average inflation rate is lower under longer quantitative easing as compared to the baseline in each of the last three sub-samples. One consequence of more deflation is a longer period of zero nominal interest rates. We will describe the economics underlying this result in more detail below but we want to mention that the basic mechanism

\[ \text{Broda and Weinstein (2007) have argued that problems in price measurement induce an upward bias of about 2 percentage points in the Japanese inflation rate. That is about the size of the average gap between our model’s average inflation rate the and the value in Japanese data.} \]
operating here is the fiscal effect we discussed in the steady-state analysis. Longer quantitative easing is associated with higher growth in money supply and with the nominal interest rate at zero the real return on money has to rise in order for households to be willing to hold it. A higher real return on money crowds out private capital and the real return on capital increases. In Table 2 we see that this effect is very persistent. The real interest under longer quantitative easing is higher than the baseline simulation in each of the final four sub-periods or a period of twenty years in total. The average magnitude of the difference is 16 basis points.

Earlier quantitative easing, induces more deflation during the 1990s but less deflation after 2001 as compared to the baseline scenario. Interestingly, this simulation shows a lower real interest rate and thus a higher wage rate than the baseline simulation. The value of the real interest rate under earlier quantitative easing is lower than the baseline in all but the first sub-sample and the real wage rate is correspondingly higher.

Table 3 reports simulation results for aggregate allocations. Quantitative easing depresses output when compared with the no quantitative easing scenario and longer quantitative easing depresses output more. Earlier quantitative easing produces higher consumption than any of the other three scenarios between 1996 and 2015 and output is higher than the other scenarios between 1996 and 2005. On net higher wages are acting to stimulate economic activity.

The fact that early quantitative easing acts to lower the real interest rate and raise output while longer quantitative easing acts to increase the real interest rate and lower output might appear to be puzzling. However, these results can be attributed to the same two distortions that we discussed in the steady-state analysis. Starting from a situation with a positive nominal interest rate, inflation acts as a tax on labor and capital. From the steady-state analysis in Table 1 we know that lower steadystate inflation rates are associated with lower monetary base growth, a lower real interest rate, higher wages and higher output. These same mechanisms are operating in the dynamic simulations. The early quantitative easing scenario exhibits lower average monetary based growth than the baseline scenario from 1991-2006 and this accounts for the fact that the earlier quantitative easing scenario has lower average inflation rates, higher output and lower real interest rates than the baseline scenario in the earlier sub-periods.

The dynamic effects of quantitative easing are quite different though once the nominal interest rate is zero. This can most readily be observed by
comparing the baseline with the longer quantitative easing scenario. The longer quantitative easing scenario exhibits higher money growth and real balances after 2001, a higher real interest rate and lower output. In the steady-state analysis above we saw that once the nominal interest rate was zero a monetary policy that increased the real return on money increased real balances and crowded out private capital.

To further explore the nature of this crowding out effect Table 4 reports the ratio of real balances to output and the capital output ratio for the four scenarios. Before discussing these results it should be pointed that in the dynamic analysis we are limiting attention to transitory changes in monetary policy. The ratio of real balances to output and the debt output ratios are the same in both the initial and terminal steady-states in all four scenarios. The results in Tables 2 and 4 indicate that this is an important distinction. Comparing the baseline scenario with the longer quantitative easing scenario, we see from Table 2 that longer quantitative easing produces more deflation. The reason for this can be seen in Table 4. Longer quantitative easing increases real balances and temporarily increases total government debt. Temporarily higher government debt increases the real interest rate and crowds out private capital. This is why the longer quantitative easing simulation exhibits lower capital output ratios, higher real interest rates, lower inflation and lower output than the baseline scenario after 2001. In other words, starting from a situation with zero nominal interest rates, the anticipated inflation effects of temporarily higher money growth are dominated by the fiscal effects of monetary policy on total government debt.

Next we turn to consider the distributional effects of quantitative easing. We allow labor productivity to vary with age. In the presence of age-specific earnings young agents face binding borrowing constraints. They would like to shift consumption forward from future periods when their labor income will be high but are unable to collateralize their future high human capital. Quantitative easing temporarily lowers taxes and this, in principal, can relax borrowing constraints. It also raises the real interest rate which reduces the incentive to consume today. These two effects can be seen in Table 5 which reports lump-sum taxes, the real interest rate and the average number of constrained cohorts for each sub-period. Notice that the baseline scenario has lower average lump-sum taxes as compared with the no quantitative easing scenario between 2001 and 2005 but that the difference is small. The

\[9^9\] We remind the reader that the period of quantitative easing was 2001-2005.
value of the real interest rate is higher during the period of quantitative easing but again the difference is small. As a result the effect of quantitative easing in relaxing borrowing constraints of the young is small. The number of cohorts that face binding borrowing constraints is a slightly lower during the period of quantitative easing (14.0 as versus 14.6 cohorts) but the pattern of borrowing constraints in other periods is very similar in the two simulations.

We observe larger differences in the number of borrowing constrained cohorts when we compare the baseline with the early and the longer quantitative easing simulations. The differences are particularly large when comparing the baseline with the earlier quantitative easing simulation. A substantially smaller number of cohorts are borrowing constrained in the first four sub-samples with earlier quantitative easing. In this simulation the young face a lower interest rate and higher lump-sum taxes but also higher wages.

The effects of quantitative easing on consumption can vary significantly with the age of the cohort. Quantitative easing benefits older individuals most. For retirees a higher real interest rate increases the value of their saving and consumption increases. Moreover, older retirees also enjoy the benefits of temporarily low taxes and escape most of the future burden of higher taxes by passing away before taxes rise. The magnitude of these benefits can be substantial. To illustrate these points we report average consumption by age in Table 6 between the years 2001 and 2005. Results are expressed as percentage consumption gains relative to the baseline and cohorts are categorized by their age as of 2001. The pattern of results for the 85+ age group indicates that they like quantitative easing and would be even happier if quantitative easing had been extended longer. Their consumption falls by 1.18 percent in the no quantitative easing scenario and by 2.23 percent in the earlier quantitative easing scenario. The old like quantitative easing because they benefit from a higher real return on their saving. Moreover, they are so old that they face a high chance of dying and thus escaping the higher taxes that occur in later years in the two quantitative easing scenarios.

The picture is reversed for middle age cohorts. They experience much higher consumption in the earlier quantitative easing scenario relative to the baseline. Those in the 55-64 age group see their consumption rise by over 2.51 percent per year in the earlier between 2001 and 2005. As can be seen in Table 5, this scenario exhibits the lowest real interest rates and thus the

\[\text{The ensuing discussion is focuses on average consumption during the period of quantitative easing and abstracts from the effects of this policy on leisure.}\]
highest wage rates of all of the scenarios.

The youngest cohorts also experience big consumption gains amounting to 1.79 percent per year in the earlier quantitative easing scenario. However, their consumption is less impacted as compared to other age groups in the other two scenarios. Their consumption increases by 0.41 percent in the no quantitative easing scenario and falls by -0.56 percent in the longer quantitative easing scenario. On net the youngest cohorts benefit more from higher wages as compared to lower taxes.

6 Conclusion

In this paper we have developed a model with real balance effects due to finite lifespans and binding borrowing constraints and used it to analyze the quantitative effects of a large increase in real balances of money in a low interest rate environment.

According to our model quantitative easing as pursued in Japan was an effective measure for limiting deflationary pressure but only had a small effect on the evolution aggregate economic activity. However, other policies that have been proposed in the literature such as early quantitative easing or longer quantitative easing have larger effects on economic activity. Our results suggest that how quantitative easing effects the economy varies depending on whether the nominal interest rate is zero. Starting from an initial situation with a positive nominal interest rate, quantitative easing lowers the real interest rate and increases the wage rate and this stimulates economic activity. Starting from a situation of a zero nominal interest rate quantitative easing has the opposite effect. It crowds out private capital, depresses wages, increases deflationary pressure and extends the period that the nominal interest rate is zero.

We also found that quantitative easing has large effects on the distribution of consumption by age. Early quantitative increases consumption of middle aged workers most and lowers consumption for the old and the young. Longer quantitative easing reduces consumption for most cohorts and only the oldest cohorts benefit.

In future work we plan to relax our current assumption that the government budget constraint is met by altering lump-sum taxes and instead make the more realistic assumption that a distortionary tax is adjusted instead. This will likely introduce stronger non-neutralities. Our model generates
borrowing and lending in equilibrium. It is consequently a good framework for modeling financial intermediation and central bank lending. In future work we plan to pursue these extensions as well.

References


Figure 1
Baseline Model and Japanese Data

K/Y

M0/(P*Y)

Per-Capita GNP

Inflation

Data
Model

Data
Model
Figure 2: M0/(P*GNP) Alternative Scenarios

- Baseline
- No Quantitative Easing
- Earlier Quantitative Easing
- Longer Quantitative Easing
<table>
<thead>
<tr>
<th>Growth rate of money (Percentage)</th>
<th>Cash Good Consumption*</th>
<th>Credit Good Consumption*</th>
<th>Money Output ratio</th>
<th>Capital Output Ratio</th>
<th>Output*</th>
<th>Real Interest Rate (Percentage)</th>
<th>Inflation Rate (Percentage)</th>
<th>Nominal Interest rate (Percentage)</th>
<th>Consumption Equiv. (percentage)</th>
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<tbody>
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<td>91.7</td>
<td>99.7</td>
<td>0.06</td>
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<td>100.0</td>
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<td>-5.0</td>
<td>0.0</td>
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</tr>
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* Cash good consumption, credit good consumption and output are expressed as a percentage of the respective variable under the Friedman rule.
<table>
<thead>
<tr>
<th>Period</th>
<th>Inflation</th>
<th>Nominal Interest rate</th>
<th>Real Interest rate</th>
<th>Wage rate</th>
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<tbody>
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<td></td>
<td>No Quantitative Easing</td>
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<td>Longer Quantitative Easing</td>
<td>Earlier Quantitative Easing</td>
</tr>
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<td>1991-1995</td>
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* The Baseline scenario reports average values of the annualized inflation rate, nominal and real interest rates and a wage rate index over the indicated sub-sample. The other results are percentage changes in each variable relative to the baseline for the indicated sub-sample.
Table 3
Simulation results: Allocations

<table>
<thead>
<tr>
<th>Period</th>
<th>Consumption</th>
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<th>Output</th>
<th>Money Growth</th>
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<tr>
<td></td>
<td>No Quantitative Easing*</td>
<td>Baseline Quantitative Easing*</td>
<td>Longer Quantitative Easing*</td>
<td>Earlier Quantitative Easing*</td>
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<tr>
<td>1991-1995</td>
<td>99.7</td>
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<td>84.9</td>
<td>97.9</td>
<td>100.8</td>
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* The Baseline scenario reports average values of annualized consumption, labor input, output and money growth over the indicated sub-sample. The other results are percentage changes in each variable relative to the baseline for the indicated sub-sample.
<table>
<thead>
<tr>
<th>Period</th>
<th>No Quantitative Easing</th>
<th>Baseline</th>
<th>Longer Quantitative Easing</th>
<th>Earlier Quantitative Easing</th>
<th>No Quantitative Easing</th>
<th>Baseline</th>
<th>Longer Quantitative Easing</th>
<th>Earlier Quantitative Easing</th>
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<tr>
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<td>0.05</td>
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<td>2.39</td>
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### Table 5: Taxes, interest rate and borrowing constraints

<table>
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<tr>
<th>Period</th>
<th>Lump-sum taxes*</th>
<th>Real interest rate</th>
<th>Number of borrowing constrained cohorts</th>
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<tr>
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*Lump-sum taxes are expressed as a fraction of total average consumption*
Table 6
Consumption age profiles relative to the baseline*

<table>
<thead>
<tr>
<th>Age in 2001</th>
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<th>Earlier Quantitative Easing</th>
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<td>35-44</td>
<td>0.50</td>
<td>-0.80</td>
<td>2.34</td>
</tr>
<tr>
<td>45-54</td>
<td>0.49</td>
<td>-0.87</td>
<td>2.31</td>
</tr>
<tr>
<td>55-64</td>
<td>0.47</td>
<td>-0.89</td>
<td>2.51</td>
</tr>
<tr>
<td>65-74</td>
<td>0.39</td>
<td>-0.59</td>
<td>2.43</td>
</tr>
<tr>
<td>75-84</td>
<td>-0.45</td>
<td>0.95</td>
<td>0.04</td>
</tr>
<tr>
<td>85+</td>
<td>-1.18</td>
<td>1.93</td>
<td>-2.23</td>
</tr>
</tbody>
</table>

*This table reports the average percentage annual consumption gain for the years 2001-2005 relative to the baseline for each of the indicated age groups.