Coping with Collapse: A Stock-Flow Consistent Monetary Macrodynamics of Global Warming

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ABSTRACT

This paper presents a macroeconomic model of growth that combines the economic impact of climate change with the pivotal role of private debt and income distribution. Using a Keen approach ([Keen, 1995]), based on the Lotka-Volterra logic, we couple its nonlinear monetary dynamics of underemployment and income distribution with abatement costs. Various damage functions reflect the loss in final production and capital due to the rise in temperature. A calibration of our model at the world scale enables us to simulate various planetary scenarios. Our findings are threefold: 1) the $+2°$C target is already out of reach, absent negative emissions; 2) the long-run dynamic consequences of climate change on economic fundamentals may lead to a severe downside. Under plausible circumstances, global warming forces the private sector to leverage in order to compensate for output and capital losses; the private debt overhang may eventually induce a global financial breakdown, even before climate change could cause serious damage to the production sector. 3) Implementing an adequate carbon price trajectory, as well as increasing the wage share, fostering employment, and reducing private debt make it easier to avoid unintended degrowth and to reach a $+2.5°$C target.

1 Introduction

Given the increasing awareness of climate disturbance, which crystallized at a diplomatic level in the Paris Agreement of December 2015, and the growing concern about potential downside consequences of a temperature increase, the question is raised of whether global warming might *per se* induce a severe breakdown of the world economy. This paper tackles this issue and looks for policies designed to mitigate climate change through a trade-off between abatement costs and a carbon price instrument at the production sector level. More particularly, at Paris, nearly 200 countries promised to try to bring global emissions down from peak levels as soon as possible. More significantly, they pledged "to achieve a balance between anthropogenic emissions by sources and removals by sinks of greenhouse gases in the second half of this century." This means getting to net zero emissions between 2050 and the end of this century. The UN climate science panel has stated that net zero emissions must happen by 2070 to avoid dangerous warming—a claim reiterated at the COP22 summit in Marrakech (2016).

Yet, beyond the bio-physical issue of warming, the financial stake of the cost of mitigation and adaptation should not be neglected either. According to the New Climate Economy Report [Economy, 2014], US$ 90 trillion are needed at the world level over the next 15 years to fund clean infrastructure that would make it possible to reach zero net emissions, which prompts a daunting question: how will the world economy finance such monetary flows? Given today's vulnerability of public finances, it is expected that the private sector will be able to endorse the much needed long-run investments. This, in turn, raises a new question: will the world economy be able to carry the corresponding additional private debt burden?[1] As argued by Bank

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[1]See [Giraud, 2017] for a first grasp of this issue.

of England Governor Mark Carney, too rapid a movement towards a low-carbon economy could materially damage financial stability: A wholesale reassessment of prospects, as climate-related risks are re-evaluated, could destabilize markets, spark a pro-cyclical crystallization of losses and lead to a persistent tightening of financial conditions: a climate Minsky moment [Carney, 2016].

Taking advantage of a growing body of literature in ecological macroeconomics —see, for instance, [Jackson, 2009], [Rezai et al., 2013], [Nordhaus and Sztorc, 2013], [Rezai and Stagl, 2016], [Dafermos et al., 2017], and [Dietz and Stern, 2015]—, we present an integrated ecological macroeconomic model that combines two sources of instability: (i) global warming and (ii) private over-indebtedness. By incorporating the latter into a rather low-dimensional stock-flow consistent integrated assessment model (hereafter IAM), we are able to track transmission channels between the two sources of potential economic breakdown alluded to by Carney. More precisely, we identify conditions under which their interplay may avoid reaching the $+2^{\circ}$ C target adopted at the 2015 Paris Agreement, or even may lead to global unintentional degrowth. Conversely, we can select carbon pricing trajectories that succeed in both avoiding a disaster and implementing a feasible cap on global warming.

The paper is organized as follows. Section 2 briefly links our stock-flow consistent model with the background literature on IAMs. Section 3 sets up the modeling framework. Section 4 provides some first mathematical insights into the destabilizing impact of climate change on our modeling. Section 5 addresses this interplay more extensively and numerically through an analysis of three prospective scenarios. Section 6 discusses the deployment of a public policy instrument—a carbon price—that may help cope with the possible climate and financial disasters that emerge from the analysis in the previous sections. Our main conclusions and areas for future research are outlined in the final section.

2 Alternative modeling foundations

Over the past thirty years, many IAMs have been developed to estimate the impact of economic development on the environment. A solid body of literature compares IAMs, describing their advantages and disadvantages, e.g., [Schwanitz, 2013]. The models considered in this literature usually involve macroeconomic settings that rely on welfare maximization, general equilibrium, partial equilibrium and cost minimization (cf. [Stanton et al., 2009]). For instance, the core economic model of DICE and the benchmark for IAM literature is the Ramsey-Cass-Koopmans approach. It assumes a closed economy endowed with a constant return to scale Cobb-Douglas production function combining labor and capital, where agents' decisions are made under perfect foresight. As a result, households perfectly anticipate the optimal budget path obtained by solving the standard intertemporal consumption-savings trade-off. At the steady state, output increases at the pace of labor force growth and technological progress while factor costs adjust such that all markets clear. By construction, such a dynamics precludes situations such as mass unemployment and over-indebtedness.

By contrast, recent research has contributed to building alternatives to such IAMs by incorporating Keynesian features (see, e.g., [Barker et al., 2012]) or more post-Keynesian insights (see, e.g. [Dafermos et al., 2017]). Our modeling approach does not assume fully optimal behavior either. Instead, it relies on the ideas of Hyman Minsky on the intrinsic instability of a monetary market economy, which have experienced a significant revival in the aftermath of the 2007–2009 financial crisis. We adopt a mathematical formalization of Minsky's standpoint to assess the role of private debt dynamics in our narrative.[2] Our starting point is the prey-predatory macrodynamics first introduced by [Goodwin, 1967] and [Akerlof and Stiglitz, 1969], and later extended by [Keen, 1995]. Building on this seminal insight, we offer a model based on the myopic behavior of imperfectly competitive firms, which is stock-flow consistent (cf. [Godley and Lavoie, 2012]), allows for multiple long-run equilibria, and exhibits endogenous monetary cycles and growth, sticky prices, endogenously determined private debt, and underemployment. Moreover, money is endogenously created by the banking sector [Giraud and Grasselli, 2016]. The non-neutrality of money enables the emergence of long-run debt-deflation, deepening the short-run phenomenon originally investigated by [Fisher, 1933][3] and whose long-term analog was recently discovered ([Grasselli and Nguyen-Huu, 2015b]). Here, by contrast with more conventional general equilibrium approaches, debt-deflation need not appear only as a "black swan"—i.e., a "rare" event relegated to the tail of risk distribution. Indeed, depending upon the basin of attraction into which the state of the economy is driven by climate damages, an ultimate breakdown may occur as the inescapable consequence of the business-as-usual (BAU) trajectory.

3 An integrated framework

Our IAM depicts the interrelations between a global monetary economy and climate disorder. Although, for simplicity, the public sector is not explicitly modeled, public policy objectives are materialized through the deployment of a

[2] [Santos, 2005] provides a survey up to 2005 of the literature on the modeling of Minskian instability; more recent contributions include [Ryoo, 2010] and [Chiarella and Di Guilmi, 2011].

[3] See also [Giraud and Pottier, 2016] for an analysis of debt-deflation within a static general equilibrium set-up with endogenously incomplete markets and money.

carbon price that allows a decentralized emission reduction rate to be achieved.[4] The core macroeconomic module absent climate change is presented in subsection 3.1 and the climate module in subsection 3.2. The introduction of damages and the way these can be controlled through public policy objectives is discussed in subsection 3.3. The empirical calibration of the IAM is provided in Appendix B.

3.1 The monetary macrodynamics

Our macroeconomic model belongs to the literature centered around [Keen, 1995].[5] One appeal of this framework lies in its ability to formalize long-term economic deflation and degrowth as a consequence of over-indebtedness.

3.1.1 Damaged production and abatement

The productive sector produces a real amount, Y^0, of a unique consumption good combining labor and capital:

$$Y^0 = \frac{K}{\nu} = aL, \tag{1}$$

where K and L refer respectively to the stock of capital and labor, while ν and a stand respectively for the (constant) capital-output ratio and Harrod-neutral labor-augmenting productivity. For simplicity, full capital use is assumed and Say's law is postulated.

As defined shortly, economic activities release CO_2-e emissions that will be priced through a carbon pricing instrument (carbon tax). As an answer to the tax burden, the productive sector may engage in abatement activities to lower its CO_2-e emissions. By doing so, a fraction, A, of output, Y^0, is diverted from its final use, and serves instead as an intermediate consumption in order to reduce CO_2-e emissions as defined shortly. Moreover, as in [Nordhaus, 2007], a proportion $\mathbf{D}^{\mathrm{Y}}$ of the remaining output is damaged beyond repair by global warming and lost. Consequently, the production available on the commodity market is given by

$$Y := (1 - \mathbf{D}^{\mathrm{Y}})(1 - A)Y^0. \tag{2}$$

3.1.2 Profits, investment and inflation

Let us denote by p the consumption price, the nominal net profit, w, the unitary money wage, r, the short-term interest rate[6], L_c, the total amount of corporate debt, M_c, the deposits held by the productive sector, p_c, the real price of a ton of CO_2-e expressed in in 2010 US\$, E_{ind}, the volume of industrial emissions in GtC, to be defined shortly, and $\delta > 0$ the standard depreciation rate of capital. Nominal profit, Π, is defined as the nominal output *minus* production cost:

$$\Pi := pY - wL - rD - pT_f - p\delta_{\mathbf{D}}K. \tag{3}$$

The cost of production is the sum of: (i) the wage bill, wL; (ii) the private debt burden—where $D := L_c - M_c$ stands for the outstanding balance of current nominal private debt; (iii) the payment of the carbon tax, $pT_f := pp_C E_{ind}$; and (iv) the global depreciation rate of capital, $\delta_{\mathbf{D}} := \delta + \mathbf{D}^{\mathrm{K}}$, where $\mathbf{D}^{\mathrm{K}}$ is the rate of degradation induced by climate change, defined shortly. Defining both the money wage share, ω, and the private debt ratio, d, by Eq. 4, the nominal profit share, π, can now be expressed by Eq. 5:

$$\omega := \frac{wL}{pY} \text{ and } d := \frac{D}{pY}, \tag{4}$$

$$\pi := \frac{\Pi}{pY}. \tag{5}$$

Real investment, I, is assumed to be driven by the profit share, π, capturing the risk appetence of the corporate sector. This leads to the following capital accumulation equation expressed in real terms:

$$I := \kappa(\pi)Y, \tag{6}$$

$$\dot{K} := I - \delta_{\mathbf{D}}K, \tag{7}$$

where $\kappa(\cdot)$ is an increasing, smooth real-valued function. The behavior of the productive sector incorporates the current level of climate damages inasmuch as investment decisions depend upon the output net of environmental damages, Y, entering into Eq. 5. For our empirical applications, the function, $\kappa(\cdot)$, introduced in Eq. 6, will be calibrated.[7]

Changes in nominal private corporate debt, D, depend on the gap between current nominal profit, Π, and nominal investment, pI, *plus* nominal dividends paid to shareholders, $\Pi_d(\pi)$:

[4] Public intervention, as well as the resulting dynamics of public debt, will be analyzed in depth in a subsequent paper.

[5] Such as [Grasselli and Lima, 2012],[Grasselli et al., 2014], [Nguyen-Huu and Costa-Lima, 2014], [Grasselli and Nguyen-Huu, 2015b] and [Giraud and Grasselli, 2016]*inter alia*.

[6] For simplicity, r is kept constant here. Endogenous short-run interest rate is left for future research.

[7] More details on our calibration are available in Appendix B. Obviously, relying on the conventional representative-agent shortcut would preclude the possibility of non-zero private debt, and hence would make little sense in our set-up. We are thus forced to consider heterogeneous corporates. We refrain, however, from providing micro-foundations to either $\kappa(\cdot)$ or $\phi(\cdot)$. Indeed, as shown by [Mas-Colell et al., 1995], when full-blown rational, profit-maximizing corporates are sufficiently numerous and heterogeneous, they are prone to an "everything-is-possible" theorem *à la* Sonnenschein-Mantel-Debreu at the aggregate level. One alternative way to recover micro-foundations without impairing the possibility of emergence phenomena would consist, e.g., in simulating some underlying agent-based model.

$$\dot{D} \quad := \quad pI + \Pi_d(\pi) - \Pi - p\delta_{\mathbf{D}}K, \tag{8}$$

$$\Pi_d(\pi) \quad := \quad \Delta(\pi)pY. \tag{9}$$

According to Eq. 9, the current level of nominal dividends, viewed as a fraction of nominal output is assumed to be an increasing real-valued function, $\Delta(\cdot)$, of the profit share, π.[8] The resulting retained profits $\Pi_r := \Pi - \Pi_d$ are used to finance various expenses as outlined in the stock-flow Table 1. Eq. 8 implies that some corporates might borrow money to finance dividends. This should not come as a surprise as it is consistent with the celebrated Modigliani-Miller theorem ([Hellwig, 1981]), which states that both equity and debt are equivalent ways to finance a corporate's expenditures. Thus, Eq. 8 may be interpreted as meaning, at least implicitly, that firms behave as if they believed the Modigliani-Miller theorem to be true. Moreover, contemporary oil companies are known to issue debt in order to pay their shareholders [Nasdaq.com, 2016].

Both the profits from the banking sector, rD, and the carbon tax, pT_f, paid by the productive sector are redistributed to the shareholders either as dividends or as a lump-sum transfer. Thus, the whole income that accrues to households is $wL + \Pi_d + rD + pT_f$. As a consequence, abatement costs are transmitted to households *via* dividends paid to shareholders: *ceteris paribus*, abatement costs reduce profits, which reduces dividends.

Finally, Eq. 10 captures the dynamics of inflation: the consumption price, p, converges to its long-run equilibrium value through a lagged adjustment of exponential form with a relaxation time, $1/\eta_p$. [9] The long-run equilibrium price is given by a markup, $m \geq 1$[10], times the average unitary cost of production, c:

$$i \quad := \quad \frac{\dot{p}}{p} := \eta_p(mc - 1), \tag{10}$$

$$c \quad := \quad \frac{1}{(1-A)(1-\mathbf{D}^{\mathrm{Y}})}\left[\frac{w}{pa} + \nu\big(\mathrm{WACC} + \delta_{\mathbf{D}}\big) + p_c\sigma(1-n)\right]. \tag{11}$$

The cost of production, c, encapsulates respectively the cost of labor, the cost of capital, and the cost of carbon emission. The WACC (weighted average cost of capital) is defined as the cost of debt services *plus* dividends, i.e., WACC:= $(rD + \Delta(\pi)pY)/pK$. The parameter, n, refers to the emission reduction rate, defined in the next section. Clearly, this dynamics of inflation (to be empirically calibrated in the sequel) presumes that the productive sector partly charges the cost of emitting CO_2-e to its clients.

3.1.3 The labor market

The world workforce, N, is assumed to grow according to a sigmoid inferred from the 15–64 age group in the United Nations median scenario[United Nations, 2015]:

$$\beta(N) := \frac{\dot{N}}{N} = q\big(1 - \frac{N}{P^N}\big), \tag{12}$$

where $P^N \approx 7.056$ billion stands for the upper bound of the world's labor force and q for the speed of convergence towards P^N.[11] The employment rate is defined in Eq. 13 as the ratio between the number of employed workers, L, and the global labor force, N:[12]

$$\lambda := \frac{L}{N}. \tag{13}$$

The link between the real and nominal spheres of the economy is provided by short-run wage-price dynamics taken from [Grasselli and Nguyen Huu, 2016].[13]

$$\frac{\dot{w}}{w} := \phi(\lambda). \tag{14}$$

In other words, workers bargain for their wages, w, based on the current state of employment, λ, through some increasing real-valued function, $\phi(\cdot)$, which will be empirically calibrated.

Finally, the behavior of households is fully accommodating in the sense that, given investment and output, consumption is pinned down by the macro balance: $C := Y - I$.

3.2 The climate module

Our formalization of the emissions and climate anomaly closely follows the conventional framework introduced by Nordhaus in his seminal work, for instance [Nordhaus, 1993] or [Nordhaus, 2014], for the DICE model, adapted here to our continuous time framework.

[8] In our empirical applications, the aggregate behavioral functions, $\kappa(\cdot)$ and $\phi(\cdot)$, have been bounded to avoid inconsistent behaviors that might fall far outside the estimation range. See [Nguyen-Huu and Pottier, 2016] for a methodological discussion of this point.

[9] The parameter η_p captures the viscosity of prices, hence it plays a role analogous to the Calvo parameter in the neo-Keynesian literature, cf. e.g., [Calvo, 1983].

[10] Whenever the consumption goods market is imperfectly competitive, $m > 1$.

[11] The details of the calibration of these parameters can be given upon request.

[12] The constraint $L \leq N$ is assumed to be never binding. Indeed, because of some irreducible frictional residual of temporary underemployment, "full employment" usually means that about 95% or so of the working population is hired. On the other hand, mass unemployment or part time labor in the Old World—and similarly, informal labor in Southern countries—make it hard to imagine economies where aggregate labor demand would be rationed.

[13] See also [Mankiw, 2010]. *Prima facie*, one might want the growth rate of money wages to depend upon inflation as well but our empirical estimations lead to a non-significant influence of i over $\dot{w}/w$ at the world level.

$$E \quad := \quad E_{ind} + E_{land}, \tag{15}$$

$$E_{ind} \quad := \quad Y^0 \sigma (1-n), \tag{16}$$

$$\frac{\dot{\sigma}}{\sigma} \quad := \quad g_\sigma, \tag{17}$$

$$\frac{\dot{g_\sigma}}{g_\sigma} \quad := \quad \delta_{g_\sigma}, \tag{18}$$

$$\frac{\dot{E}_{land}}{E_{land}} \quad := \quad \delta_{E_{land}}. \tag{19}$$

As shown by Eqs 15–19, global CO_2-e emissions are the sum of: (i) industrial emissions, E_{ind}, linked to real produced output and (ii) land-use emissions, E_{land}.[14] The latter source of emissions is exogenous and decreases at rate $\delta_{E_{land}} < 0$. The level of industrial emissions defined in Eq. 16 depends on the current emission intensity of the economy, σ,[15] mitigation efforts captured through the emission-reduction rate, n, defined shortly, and the output level of the economy.

$$\begin{pmatrix} \dot{\text{CO}_2}^{AT} \\ \dot{\text{CO}_2}^{UP} \\ \dot{\text{CO}_2}^{LO} \end{pmatrix} \quad := \quad \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} + \Phi \begin{pmatrix} \text{CO}_2^{AT} \\ \text{CO}_2^{UP} \\ \text{CO}_2^{LO} \end{pmatrix}, \tag{20}$$

where:

$$\Phi \quad := \quad \begin{pmatrix} -\phi_{12} & \phi_{12} C_{UP}^{AT} & 0 \\ \phi_{12} & -\phi_{12} C_{UP}^{AT} - \phi_{23} & \phi_{23} C_{LO}^{UP} \\ 0 & \phi_{23} & -\phi_{23} C_{LO}^{UP} \end{pmatrix} \tag{21}$$

$$C_i^j \quad := \quad \frac{C_{j_{pind}}}{C_{i_{pind}}}, (i,j) \in \{AT, UP, LO\}^2, \tag{22}$$

The carbon cycle is represented in Eqs 20–22 through an interacting three-layer model in which global CO_2-e emissions, E, accumulate, including: (i) the atmosphere (AT); (ii) a mixing reservoir in the upper ocean and the biosphere (UP); and (iii) the deep ocean (LO). When the level of emissions disappears following completion of the energy shift, the total amount of CO_2-e (existing and released) will spread according to the diffusion parameters $\Phi_{ij}, (i,j) \in \{1,2,3\}^2$, such that the relative preindustrial concentrations $C_{i_{pind}}, i \in \{AT, UP, LO\}$ in each layer are respected at equilibrium.

$$F \quad := \quad F_{ind} + F_{exo}, \tag{23}$$

$$F_{ind} \quad := \quad \frac{F_{2\times CO_2}}{\log(2)} \log\left(\frac{CO_2^{AT}}{C_{AT_{pind}}}\right), \tag{24}$$

The accumulation of greenhouse gases modifies the chemical properties and thus the energy balance of the atmosphere layer, triggering a rise in the radiative forcing, F, of CO_2-e as modeled by Eqs 23–24. A distinction is made between industrial forcing, F_{ind} (from CO_2-e) and residual forcing, F_{exo}.[16] Note that in Eq. 24, the parameter $F_{2\times CO_2}$ represents the increase in the radiative forcing resulting from a doubling of the preindustrial CO_2-e concentration.

$$C\dot{T} \quad := \quad F - \rho T - \gamma^*(T - T_0), \tag{25}$$

$$C_0 \dot{T}_0 \quad := \quad \gamma^*(T - T_0). \tag{26}$$

Eventually, the rise in radiative forcing induces a change, T, in the global mean atmospheric temperature, as follows from Eqs 25–26. The global thermal behavior results from a coupled two-layer energy-balance model that roughly represents: (i) the atmosphere, land surface, and upper ocean with a mean temperature, T, and (ii) the deeper ocean with a mean temperature, T_0. In this framework, the latter layer encapsulates the long-run thermal inertia effects of the climate system. The remaining parameters are: ρ, the radiative feedback parameter; γ^*, the heat exchange coefficient between the two layers; C, the heat capacity of the atmosphere, land surface, and upper ocean layer; and C_0, the heat capacity of the deep ocean layer. As [Geoffroy et al., 2013] pointed out, this formalism enables us to account for the two-frequency responses of the mean atmospheric temperature change through a distinct transient climate response (TCR) and an equilibrium climate sensitivity (ECS, determined by $T = F/\rho$ in this framework).[17].

3.3 Damages and mitigation

We couple the macroeconomic and climate modules in a way similar to [Nordhaus, 2014], that is, through: (i) an

[14] This second contribution can be viewed as being induced by deforestation and the implied release of CO_2-e.

[15] The dynamics of σ is given by Eqs 17 and 18, where $\delta_{g_\sigma} < 0$ is a parameter controlling the exogenous decay of emission intensity.

[16] The residual forcing results from various residual factors such as non-CO_2-e long-lived greenhouse gases and other factors such as albedo changes, or the cloud effect. For simplicity, it is taken here as exogenous, as [Stocker et al., 2013] showed it to be negligible and in line with representative concentration pathways. To do so, we rely on the formalization of [Nordhaus, 2017] with a linear trajectory up to 2100, followed by a plateau.

[17] TCR and ECS represent the mean atmospheric temperature deviations, at different time scales, induced by the change of radiative forcing resulting from a linear doubling of the atmospheric CO_2-e concentration (at a 1% increase rate of the stock *per annum*, hence a doubling in about 70 years). The TCR denotes the deviation obtained at the end of this doubling, while the ECS accounts for the new equilibrium of the system, reached decades later due to its thermal inertia. In our setting, assuming the calibration given in Appendix B, we find a TCR of approximately 1.5, which is in line with the Fifth Assessment Report [Stocker et al., 2013]

environmental damage function that quantifies the real economic loss due to global warming and (ii) mitigation efforts implemented through the deployment of a carbon price instrument. Finally, achieving the energy shift will impose a carbon abatement cost, already introduced above.

3.3.1 Environmental damages

The damage function summarizes the total economic impacts brought on by the rise in mean atmospheric temperature. It thus has to compile a wide range of events, including biodiversity loss, ocean acidification, sea level rise, change in ocean circulation, and highly frequent storms, among others, and consequently exhibits highly nonlinear and threshold effects. Conventional damage functions, as introduced by Nordhaus in his seminal work [Nordhaus and Sztorc, 2013], are designed to express the aggregate economic impact of climate change as a fraction of current real output. However, as rightly pointed out by [Dietz and Stern, 2015] and [Dafermos et al., 2017], global warming may have an adverse impact not only on output but also on the factors of production themselves, such as the capital stock. To capture the total amount of damages, $\mathbf{D}$, on the economy, and the way it distributes between output, $\mathbf{D}^{\mathrm{Y}}$, and the stock of capital, $\mathbf{D}^{\mathrm{K}}$, we borrow the functional form and calibration provided by [Dietz and Stern, 2015] with a polynomial damage function as described in Eq. 27

$$\mathbf{D} := 1 - \frac{1}{1+\pi_1 T + \pi_2 T^2 + \pi_3 T^{\zeta_3}}, \tag{27}$$

$$\mathbf{D}^{\mathrm{K}} := f_K\,\mathbf{D}, \tag{28}$$

$$\mathbf{D}^{\mathrm{Y}} := 1 - \frac{1-\mathbf{D}}{1-\mathbf{D}^{\mathrm{K}}}. \tag{29}$$

The allocation rule given in Eqs 28 – 29 ensures that the *instantaneous* level of damages on real output is identical to the one introduced by [Dietz and Stern, 2015]. In the medium run, however, our specification will have more severe effects since the potential output of the economy, driven by the current stock of capital, is now penalized.

3.3.2 Abatement efforts and reduction of emissions

Carbon emission abatement is achieved at some cost, which is borne by the productive sector as an intermediary consumption. A fraction of real output, A, is diverted from sales in order to reduce the burden of the carbon tax, which is calculated upon the total level of industrial CO_2-e emissions. Depending on the level of carbon price, the productive sector endogenously chooses its emission reduction rate, $n \geq 0$, which depends on the absolute value of the carbon tax, as well as on the abatement technology, A. The latter involves the emission intensity, σ,[18] and the price of the backstop technology, p_{BS}:

$$A := \frac{\sigma p_{BS}}{\theta} n^{\theta}, \tag{30}$$

where the parameter θ controls the convexity of the cost.

The resulting emission reduction rate, n, is the outcome of an arbitrage between the carbon price p_C[19] and the backstop technology price, p_{BS}:[20]

$$n := \min\left\{ \left(\frac{p_c}{p_{BS}}\right)^{\frac{1}{\theta-1}} ; 1 \right\}. \tag{31}$$

The backstop technology is available at a passively declining price:

$$\frac{\dot{p}_{BS}}{p_{BS}} := \delta_{p_{BS}} \leq 0. \tag{32}$$

As for the carbon price, this will be treated as an exogenous variable whose exponential trajectory drives our various scenarios:

$$\frac{\dot{p}_C}{p_C} := \delta_{p_C}(\cdot) \geq 0. \tag{33}$$

Section 6 provides a numerical prospective analysis of the impact of such carbon trajectories.

3.4 Stock-flow consistency

Table 1 displays the stock-flow consistency of our model. It can be readily checked, in particular, that the accounting identity "investment = saving" always holds.

[18] As previously mentioned, the energy intensity, σ, passively declines, slowly improving the environmental performance of the economy.

[19] p_C refers to the price per ton of CO_2-e.

[20] For the sake of clarity, n can be seen as the solution of a cost-minimization program between the abatement cost, AY, on the one hand, and the carbon tax, $p_C E_{ind}$, on the other hand.

	Households	Productive Sector		Banks	Sum
Balance Sheet					
Capital stock		pK			pK
Deposits	M^h	M^c		$-M$	
Loans		$-L_c$		L_c	
Equities	E	$-E^f$		$-E^b$	
Sum (net worth)	X^h	$X^f = 0$		$X^b = 0$	X
Transactions		current	capital		
Consumption	$-pC$	pC			
Investment		pI	$-pI$		
Acc. memo [GDP]		$[pY]$			
Wages	W	$-W$			
Capital depr.		$-(\delta + \mathbf{D}^{\mathrm{K}})pK$	$(\delta + \mathbf{D}^{\mathrm{K}})pK$		
Carbon taxes	pT_f	$-pT_f$			
Int. on loans		$-r_c L_c$		$r_c L_c$	
Bank dividends	Π_b			$-\Pi_b$	
Productive sector dividends	Π_d	$-\Pi_d$			
Int. on deposits	$r_M M^h$	$r_M M^c$		$-r_M M$	
Column sum (balance)	S^h	Π_r	$-pI + (\delta + \mathbf{D}^{\mathrm{K}})pK$	S^b	
Flow of Funds					
Change in capital stock		$p\dot{K}$			$p\dot{K}$
Change in deposits	$\dot{M}^h$	$\dot{M}^c$		$-\dot{M}$	
Change in loans		$-\dot{L}_c$		$\dot{L}_c$	
Column sum (savings)	S^h	Π_r		S^b	
Change in equities	$\dot{E}^f$	$-(\Pi_r + \dot{p}K)$			
Change in bank equity	$\dot{E}^b$			$-S^b$	
Change in net worth	$S^h + \dot{E}$	0		0	$\dot{p}K + p\dot{K}$

Table 1: Balance sheet, transactions, and flow of funds in the economy

The monetary counterpart of our economy can now be made explicit: M stands for total deposits and equals M^h, the deposits of households, *plus* M^f, the deposits of the productive sector. Since dividends of both financial and non-financial entities are redistributed to households, the latter own both types of equities, resp. E^f and E^b. Notice that, since the banks' financial balance is always zero, their equity, E^b, can safely be assumed constant. Similarly, we assume that the market value of the productive sector's equity is constant (say, because stock markets are closed in this model). Moreover, it follows from Eq. 8 and the accounting identity

$$pY = \Pi + W + rD + pT_f + \delta_{\mathbf{D}} pK = pC + pI$$

that $W + \Pi_d + rD + pT_f = \dot{D} + pC$. Consequently, $\dot{M}^h = \dot{D} = \dot{L} - \dot{M}^f$: the change in corporate debt equals the change in households' saving.

4 Long-run analysis

To begin to understand how climate affects the path of the world economy, this section analyses the long-term properties of our macroeconomic system, with a focus on the asymptotic stability of its steady states.[21]

4.1 Reduced-form after the energy shift

The model presented in Section 3 boils down to a 16-dimensional nonlinear dynamical system. The economic and climate modules are coupled through: (i) emissions defined in Eq. 15 and (ii) damages specified in Eqs 27, 28, and 29. Once the energy shift is fully completed, there are no longer any additional emissions. As a consequence, once carbon neutrality is achieved, the mean atmospheric temperature deviation, T_{eq}, remains constant, while abatement efforts vanish: $A_{eq} = 0$.[22] The only remaining exogenous term of the climate module, F_{exo}, follows a linear path until 2100 where it reaches a plateau. Its upper limit is thus reached at every long-run equilibrium. As a result, damages, $\mathbf{D}^{\mathrm{Y}}$ and $\mathbf{D}^{\mathrm{K}}$, converge towards finite limits, ${\mathbf{D}^{\mathrm{Y}}}_{eq}$ and ${\mathbf{D}^{\mathrm{K}}}_{eq}$. Therefore, an analysis of the macroeconomic module at its long-run equilibrium (if any) can be performed.

The macroeconomic module reduces to the following

[21] By steady state, we mean, as usual, a state of the economy where the variables driving the reduced form system—namely the wage share, the employment rate, the private debt ratio, and the global workforce—remain constant, while auxiliary variables—such as real growth or inflation—rise at a constant pace.

[22] Once the energy shift is achieved, the emission reduction rate equals one, whereas both the energy intensity and the price of the backstop technology are null. As a result, abatement efforts vanish, see Eq. 30.

system of non-linear differential equations:

$$\begin{cases} \dot{\omega} &= \omega\left[\phi(\lambda) - i - \alpha\right] \\ \dot{\lambda} &= \lambda\left[g - \alpha - \beta(N)\right] \\ \dot{d} &= -d\big(g+i\big) + \kappa(\pi) + \Delta(\pi) - \pi - \frac{\nu\delta_{\mathbf{D}}}{1-\mathbf{D}^{\mathrm{Y}}} \\ \dot{N} &= \beta(B)N \end{cases} \tag{34}$$

with, as auxiliary variables, the profit share, the unitary cost of production, as well as the growth rates of the population, the consumption price, and of the real output:

$$\begin{aligned} \pi &= 1 - \omega - rd - \left(\frac{p_C\sigma + \delta_{\mathbf{D}}\nu}{1-\mathbf{D}^{\mathrm{Y}}}\right), \\ c &= \omega + rd + \Delta(\pi) + \frac{\nu\delta_{\mathbf{D}}}{1-\mathbf{D}^{\mathrm{Y}}}, \\ \beta(N) &= q\left(1 - \frac{N}{P^N}\right), \\ i &= \eta_p(mc-1), \\ g &:= \frac{\dot{Y}}{Y} = \frac{\kappa(\pi)(1-\mathbf{D}^{\mathrm{Y}})}{\nu} - \delta_{\mathbf{D}}. \end{aligned}$$

4.2 Long-term equilibria with no inflation

To make some progress in the analysis, it is useful to first assume no inflation ($i = 0$). This restriction will of course be relaxed for our numerical analysis in Section 5, performed with the full-blown calibrated model. The system (34) then admits a "Solovian" equilibrium, characterized by a balanced growth path, whose real growth rate is $g = \alpha$ — as in a standard Solow or Ramsey growth model.[23] At this long-run steady state, the profit rate is given by

$$\pi_{eq} = \kappa^{-1}\left(\nu\frac{\alpha + \delta(\mathbf{D}^{\mathrm{K}}{}_{eq})}{1-\mathbf{D}^{\mathrm{Y}}{}_{eq}}\right).$$

As a consequence, an increase of the mean atmospheric temperature deviation will lead to higher investments, namely to the level needed to keep the economy on the balanced growth path despite damages induced by global warming. This increase will translate into a rise in the steady-state value of profit share, π_{eq}, and of the debt-to-output ratio, d_{eq}. This first channel makes explicit the destabilizing impact of warming, since more debt relative to output means that the economy is more prone to some Minskian financial instability. The balanced growth path is characterized by

$$\begin{cases} \omega_{eq} &= 1 - \pi_{eq} - rd_{eq} - \frac{\nu\delta(\mathbf{D}^{\mathrm{K}}{}_{eq})}{1-\mathbf{D}^{\mathrm{Y}}{}_{eq}}, \\ \lambda_{eq} &= \phi^{-1}(\alpha), \\ d_{eq} &= \frac{\kappa(\pi_{eq}) + \Delta(\pi_{eq}) - \pi_{eq} - \frac{\nu\delta(\mathbf{D}^{\mathrm{K}}{}_{eq})}{1-\mathbf{D}^{\mathrm{Y}}{}_{eq}}}{\alpha}, \\ N_{eq} &= P^N. \end{cases} \tag{35}$$

Furthermore, the combined effects of the rise of the profit rate just alluded to and of the debt ratio will penalize the wage share, as they mechanically reduce the amount of remaining undistributed income. More precisely, ω_{eq} can be rewritten as:

$$\omega_{eq} = 1 - \frac{\alpha - r}{\alpha}\pi_{eq} - r\frac{\nu\frac{\alpha}{1-\mathbf{D}^{\mathrm{Y}}{}_{eq}} + \Delta(\pi_{eq})}{\alpha}. \tag{36}$$

Since the profit share, π_{eq}, increases as damages become more severe, a sufficient condition for the wage share to be reduced in the long run by global warming is that the equilibrium growth rate, α, be greater than the interest rate, r.[24] Finally, the long-run employment rate, λ_{eq}, is a monotonic function of the steady-state real growth rate—a direct consequence of the short-term Phillips curve, see Eq. 14.

Turning to the local asymptotic stability of this balanced growth path, the partial derivative of $\dot{d}$ with respect to ω at equilibrium leads to the following necessary and sufficient condition:

$$r\left[\left(\frac{d_{eq}}{\nu}(1-\mathbf{D}^{\mathrm{Y}}{}_{eq}) - 1\right)\kappa^{'}(\pi_{eq}) + \Big(1 - \Delta'(\pi_{eq})\Big)\right] < 0. \tag{37}$$

Details of its computation are given in Appendix A. A little algrebra shows that, whenever $r > 0$ and ω decreases, local asymptotic stability of the balanced growth path requires d to increase, and vice-versa. This is broadly in line with what has been observed in the last decades in Western countries. This reassuring conclusion, however, needs to be confirmed when both inflation and climate change enter the picture. As we shall now see, the global stability of the system weakens considerably with climate change.

5 Prospective analysis

We now turn to a numerical analysis of the impact of climate disorder along the transition path of the economy towards one of its steady states. By contrast with the previous section, this time we take due account of inflation. Let us first explore our four main scenarios, chosen for their illustrative qualities.

[23] In our set-up, the workforce growth parameter, β, is null at the equilibrium courtesy of the demographic transition.

[24] This might come as a surprise at first glance, since $g > r$ has been popularized as being the hallmark of increasing inequality in favor of capital. First, [Giraud and Grasselli, 2016] show that this alleged symptom is actually misleading. Second, here, r is the short-run interest rate, and is not necessarily equal to the return on capital. Last, along the transitional dynamics and absent damages, $\omega = 1 - \pi - rd$, so that Eq. 36 captures but the relationship between ω and π at the Solovian steady state. Of course, this relationship will have to be re-examined with an endogenous interest rate, set, for instance, by the central bank, as a function of inflation. We leave this for further research.

5.1 The four main scenarios

We calibrated our macroeconomic module at the world level using data from the World Bank, Penn University, the U.S. Bureau of Economic Analysis, and the United Nations,[25] while the climate module was calibrated on the Nordhaus DICE 2016R model through a method of moments. Appendix B reports the details of this calibration, and Appendix C provides the initial values of our integrated dynamics. The path of the world economy is simulated over the period 2016–2300 for the study of steady states—in order to account for the long-term inertial effects of climate on the economy—but we shall focus our inquiry of the transitional dynamics over the range 2016–2100 in line with the temporal horizon of climate policymakers.

We consider four classes of scenarios. First comes the *Baseline scenario*, which is a business-as-usual trajectory *with no* climate feedback loop. This thought experiment will provide a macroeconomic benchmark, absent climate considerations. Second, the *Type 1 scenario* introduces Nordhaus-type climate damages—where the functional form of the damage function is that of [Nordhaus, 2007], and damages are allocated only to the flow of production. It allows for a qualitative comparative analysis with the DICE model.[26] Third, and in line with the recent literature ([Dietz and Stern, 2015], [Lenton et al., 2008]), the *Type 2 scenario* refines the impact of climate change, using the same damage function as Type 1 scenario, but allocated to both output and capital. Fourth, the *Type 3 scenario* takes advantage of [Weitzman, 2012] by introducing an alternative and more convex functional form of climate damages. Table 2 wraps up our four classes of scenarios.[27]

Scenario	**Baseline**	**Type 1**	**Type 2**	**Type 3**
Damage Type	-	Nordhaus	Nordhaus	Weitzman
Damage on output	-	Yes	Yes	Yes
Damage on capital	-	-	Yes	Yes

Table 2: The four scenarios.

5.2 The baseline case

Figure 1 below presents the trajectory obtained in the Baseline case and Table 3, some of its key figures. The (exogenous) deterministic exponential growth of productivity, a, feeds the rise of real GDP, which in 2100 reaches eight times its initial 2015 volume. At variance with [Goodwin, 1967], the trajectories obtained in Figure 1 display few oscillations. This is the consequence both of the introduction of private debt (which turns the conservative dynamical system into a dissipative one) and inflation. This confirms a remark already made by [Grasselli and Nguyen-Huu, 2015a]: inflation has a dampening effect on the endogenous real business cycles of the underlying Lotka-Volterra dynamics. Here, inflation starts at approximately 0.5%, and converges in the range of 2–2.5% *per annum*. The long-run NAIRU[28] oscillates around 30%—close to the current world average rate of unemployment. The wage share converges in the vicinity of 60%, and the debt-to-output ratio stabilizes around 180% (slightly above its current level). In other words, absent any climate impact, the world economy would converge towards a quite reasonable balanced growth path. Nonetheless, the collapse, where $(\omega, \lambda, d) \to (0, 0, +\infty)$, remains a possible steady state as well. And it is worth mentioning that, given our empirical estimation, both this "bad" attractor and the Solovian steady state alluded to earlier turn out to be locally asymptotically stable. What Figure 1 actually shows is therefore that our world economy belongs to the basin of attraction of the desirable long-run equilibrium. [29]

[25] More precisely, the behavioral aggregate functions (i.e., the Phillips curve, investment, and dividends) were empirically estimated, while the remaining parameters were calibrated. See also footnote 7. Further details about our methodology are available upon request.

[26] Both models differ in their macroeconomic cores and initial values but share identical climate module, climate feedback loop, and allocation of damages. Otherwise stated, DICE and our model under the Type 1 scenario differ only through their macrodynamics. The next scenarios introduce damages that differ from DICE.

[27] Of course, a number of other scenarios are conceivable—e.g., by coupling an endogenous temperature-driven productivity with a given damage function, etc. They are available upon request.

[28] It is easy to see, indeed, that our structural unemployment rate, $1 - \overline{\lambda}_1$, is close to the NAIRU ("non-accelerating inflation rate of unemployment") introduced in [Tobin, 1980]: at the balanced growth path, inflation, $i(\overline{\omega}_1)$, remains constant. As for the NIRU ("non-inflationary rate of unemployment") of [Modigliani and Papademos, 1975], this is given by $\lambda_1^* := \Phi^{-1}(\alpha)$ and corresponds to the special case of no-inflation in the long run, which obtains, e.g., for $m = 1/c$.

[29] Notice, nevertheless, that Figure 1 exhibits a sharp increase of CO_2-e emission, resulting in an increase of the temperature anomaly of approximately $+10°$C. As we shall see, including climate back-loop will have a dramatic effect.

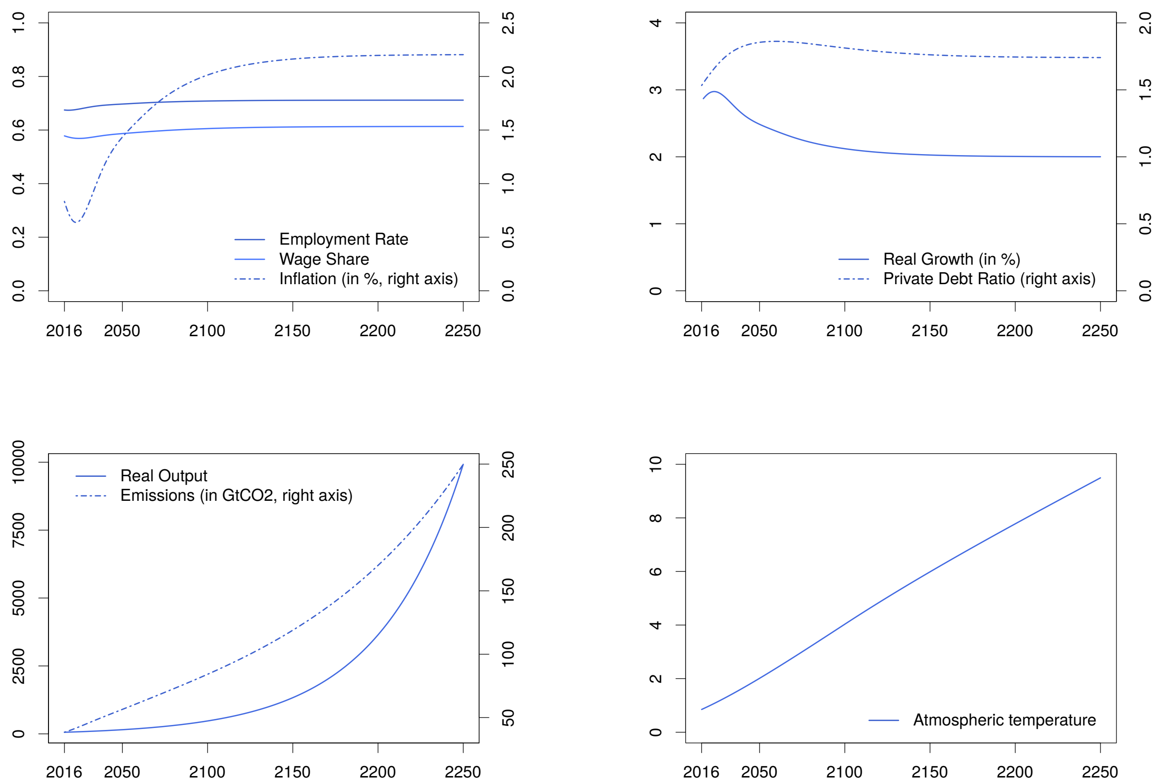

Figure 1: Trajectories of the main simulation variables absent climate change and climate policy.

On the other hand, we saw previously that climate damages tend to reduce the wage share. Since, for simplicity, in this paper inflation is cost-push (i.e., solely driven by money wages), global warming will lower inflation and therefore increase the level of debt-to-GDP ratio. Eventually, climate change will thus 1) offset the dampening effect of inflation, favoring destabilizing oscillations, and 2) increase the Minskian vulnerability of the entire economy by fostering the rise of d. Can these effects distract the world economy from converging towards the balanced growth path?

5.3 Long-term analysis

To address this question, and echoing the discussion of subsection 4.2, let us first consider how the long-run Solovian equilibrium is affected by global warming in the phase diagram (ω, λ, d). In this subsection, we therefore treat the temperature anomaly as an exogenous parameter (independent of abatement costs and emissions) and plot the corresponding long-run balanced growth path. To do so, we used the world calibration detailed in Appendix B and C, to simulate the system (34) together with an exogenously given temperature deviation (and its associated damages). We then numerically identified the long-run equilibrium. In the following subsection, we provide a geometric view of the destabilizing effect of global warming through its influence on the basin of attraction of the long-term steady state.

5.3.1 The Solovian steady state as a function of temperature

For the sake of clarity, consider first environmental quadratic damages on output (Type 1 scenario) and, second, more stringent environmental damages both on output and capital (Type 3 scenario).

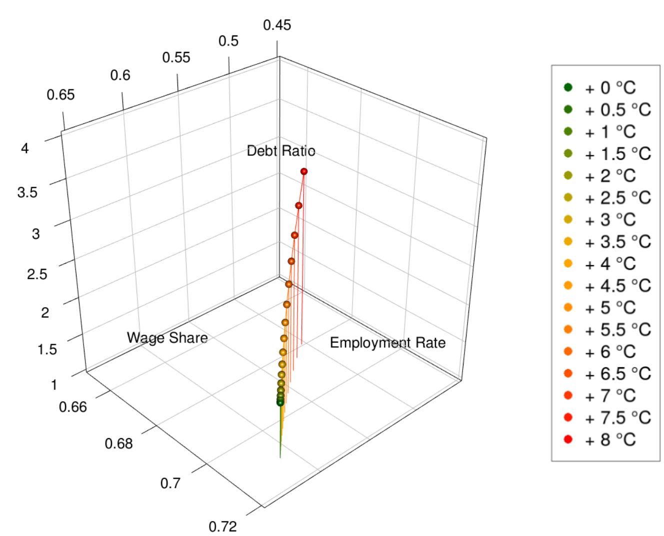

Figure 2: Perturbation of the Solovian steady state parameterized by temperature in the Type 1 scenario.

As shown by Figure 2,the long-run equilibrium of the Type 1 scenario moves along a quasi-affine trajectory, parameterized by temperature anomaly.

In line with the analysis performed in subsection 4.2, as the temperature anomaly rises, the long-run private debt ratio increases while the wage share declines. As a result, the debt burden fuels a growing financial fragility which, as we shall see, will ultimately prevent the economy from following the balanced growth path.

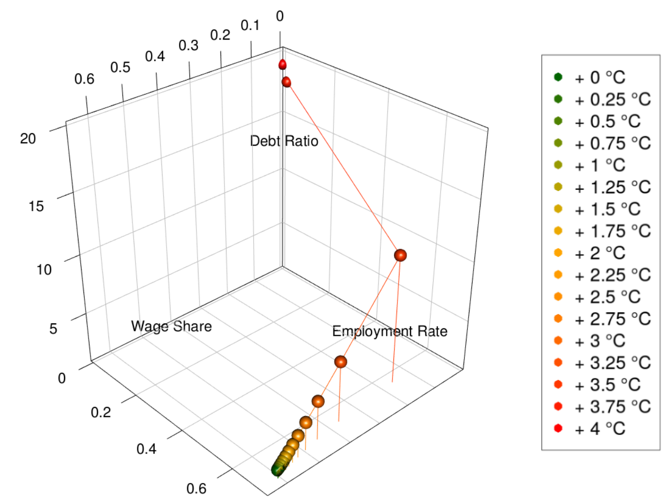

Figure 3: Perturbation of the Solovian steady state parameterized by temperature in the Type 3 scenario.

We now turn to the Type 3 scenario. As shown by Figure 3, when the temperature rises, the economy first keeps following the balanced growth path in the same way as in Scenario 1. Between + 3°C and +4°C, however, a bifurcation occurs, after which the world economy ends up in a catastrophic attractor where damages induced by global warming can no longer be offset: zero wage share and employment rate, unbounded debt ratio. This confirms [Lenton et al., 2008], who concludes using completely different methods that the +4°C threshold may indeed be a tipping point for the climate system. Our simulations further suggest that, prior to this tipping point, the long-run employment rate should be close to 70%, the equilibrium wage share should shrink to 50%, and private debt should equal approximately ten times the world GDP.

5.3.2 A geometric view of the destabilizing effect of warming

Climate change impacts the macro-dynamics of the world economy inasmuch as it makes the Solovian long-run equilibrium more difficult to reach. A bifurcation occurs whenever the "climate perturbation" is strong enough to prevent the economy from converging towards this desirable steady state. In terms of public policy, the trouble with this conclusion is 1) that the modifications induced by climate change on the Solovian equilibrium highlighted in Figure 3 are a long-term phenomenon that may not be noticeable in the short run—it might be that our world economy is already following a path towards a bad attractor without exhibiting much difference, in the short-term, from trajectories that would lead to a balanced growth path. Then 2), the rise in temperature is treated as an exogenous variable whereas it in fact depends upon the actual path followed by the economy. This prompts two questions: can we infer the long-run fate of our economy from short-run information? And is the kind of bifurcation just highlighted still to be expected once global warming is endogenized?

We address these questions from a geometric perspective by comparing the basins of attraction of the desirable steady state with and without climate change. From this subsection on, and by contrast with subsection 5.2, the whole apparatus of climate feedbacks linking the world economy to our climate module is taken into account, so that the final temperature at the long-run steady state does depend upon the path followed by the economy. Varying initial conditions will possibly lead to various emission paths, hence to different equilibrium temperature deviations and eventually to different "good" equilibria (whenever the latter are still attainable). Given some specified emission-reduction rate path, we shall consider the set of *all* initial conditions that do *not* lead to an economic cave-in. Let us call this set the "good" basin of attraction.

In order to numerically approximate the "good" basin of attraction, we started with a reasonable range of initial conditions for the variables of interest (wage share, employment rate, debt ratio), outside of which the world economy is definitely not viable.[30] We then considered another compact set to which long-term solutions had to belong in order to be considered as economically desirable (hereafter the "convergence set").[31] Any long-run steady state outside of this convergence set could not justifiably be claimed to be a "good" equilibrium. Finally, we assumed a common emission-reduction rate path for the world economy defined here as the minimal path avoiding a collapse, given the postulated initial conditions. More precisely, we considered an initial real carbon price of 2010 US$ 2 t/CO_2-e_2 in 2016, in line with the calibration of the backstop technology path, and its associated minimal exponential growth necessary to avoid a collapse (under the initial conditions presented in Appendix C). We then computed the trajectory starting anywhere in the initial set and checked whether it ended up in the convergence set at a large time scale. Whenever this was the case, the starting point was then considered to be

[30] Our initial set is: $(\omega, \lambda, d) \in [0.2 : 0.99]^2 \times [0.1 : 2.7]$. The choice of 2.7—the constant capital-to-output ratio—is a threshold value so that if $d > 2.7$, we have $D > pK$. In other words, the level of nominal debt is higher that the market value of capital. Such a situation will most likely lead to a substantial amount of bankruptcy not modeled in this paper, since existing capital is the only available collateral for debt, see [Fostel and Geanakoplos, 2015].

[31] The convergence set is: $(\omega, \lambda, d) \in [0.4 : 0.99]^2 \times [0.1 : 2.7]$.

part of the "good" basin of attraction.

We carried out this thought experiment assuming alternatively (i) no climate change (zero emissions) or (ii) environmental damages of the Type 1 scenario. Figure 4 first plots the "good" basin of attraction obtained with no global warming.

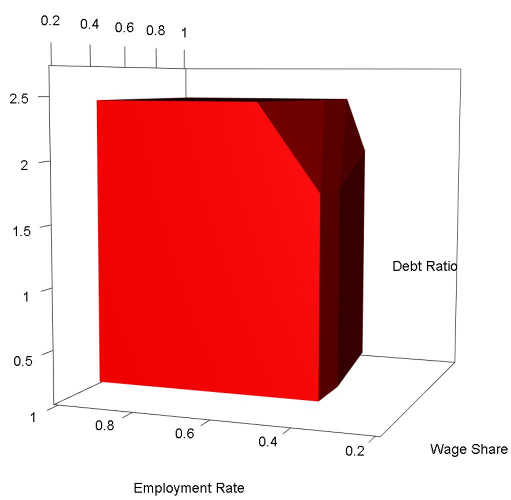

Figure 4: "Good" basin of attraction without climate change.

It turns out that almost all (reasonable) initial conditions lead to the convergence set. This emphasizes the robustness of the conclusions reached in subsection 5.2 *supra* regarding the BAU scenario: absent climate change, our modeling approach seems to convey an optimistic narrative in which the world economy converges to some rather desirable long-run steady state almost independently of its starting point. Figure 5, by contrast, displays the "good" basin of attraction obtained in the Type 1 scenario.[32]

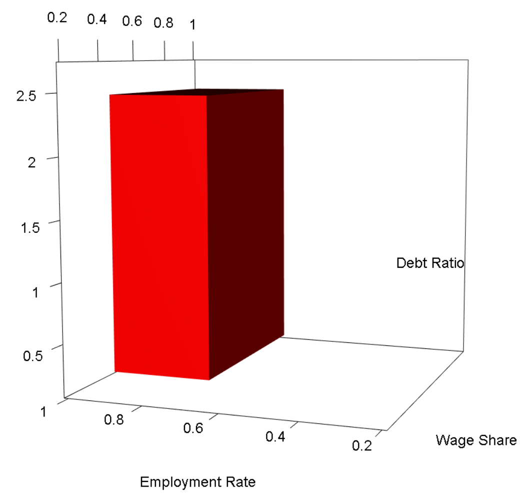

Figure 5: "Good" basin of attraction in the Type 1 scenario.

Global warming obviously narrows the set of initial conditions that allow our world economy to avoid an economic collapse. Interestingly enough, the higher the wage share today and the higher the employment rate, the easier it will be for the world economy to circumvent a disaster. This is not surprising given the positive impact (already mentioned) of climate damages on the profit share: the need to invest more so as to compensate for losses in output and capital will favor the distribution of wealth towards capital holders. One way to compensate for this trend is to have an initial distribution biased in favor of labor. Since π is only a ratio, this, of course, does not imply that investors will be better off with climate change than without, especially when real output collapses (as we shall see in the next section). Nonetheless, to the best of our knowledge, Figs 4 and 5 provide the first analytical link between climate change and the classical capital-labor distribution of wealth issue: a more resilient economy is one where labor is favored. Notice as well that resilience against global warming also calls for less private debt: an initially over-indebted economy turns out to be incapable of carrying the burden of new debt resulting from additional investment triggered by climate destruction.

5.4 Transitional dynamics

We are now ready to plunge into the details of our prospective scenarios with global warming. For each non-baseline scenario presented in subsection 5.1, we first examine the trajectory obtained together with a rather mild emission-reduction rate path, close to the one considered by [Nordhaus and Sztorc, 2013]. This policy is based on a real carbon price fixed in 2016 at an initial value of 2010 US$ 2 t/CO_2-e_2,[33]

[32] The computation of the "good" basins of attraction for other types of damage and allocation leads to similar results, namely, that as damages become more severe, the set of acceptable initial conditions under which a collapse may be avoided shrinks dramatically.

[33] This value is chosen so as to be compatible with an initial emission reduction rate of 3% and the initial condition for the price of the backstop technology. Again, the details of the calibration are given in Appendix B and C, together with an average growth rate of 2% per year. Let us first assess the various prospective narratives that emerge from the set of scenarios we can envisage. Figure 6 and Table 3 allow us to draw a first comparison between our scenarios.

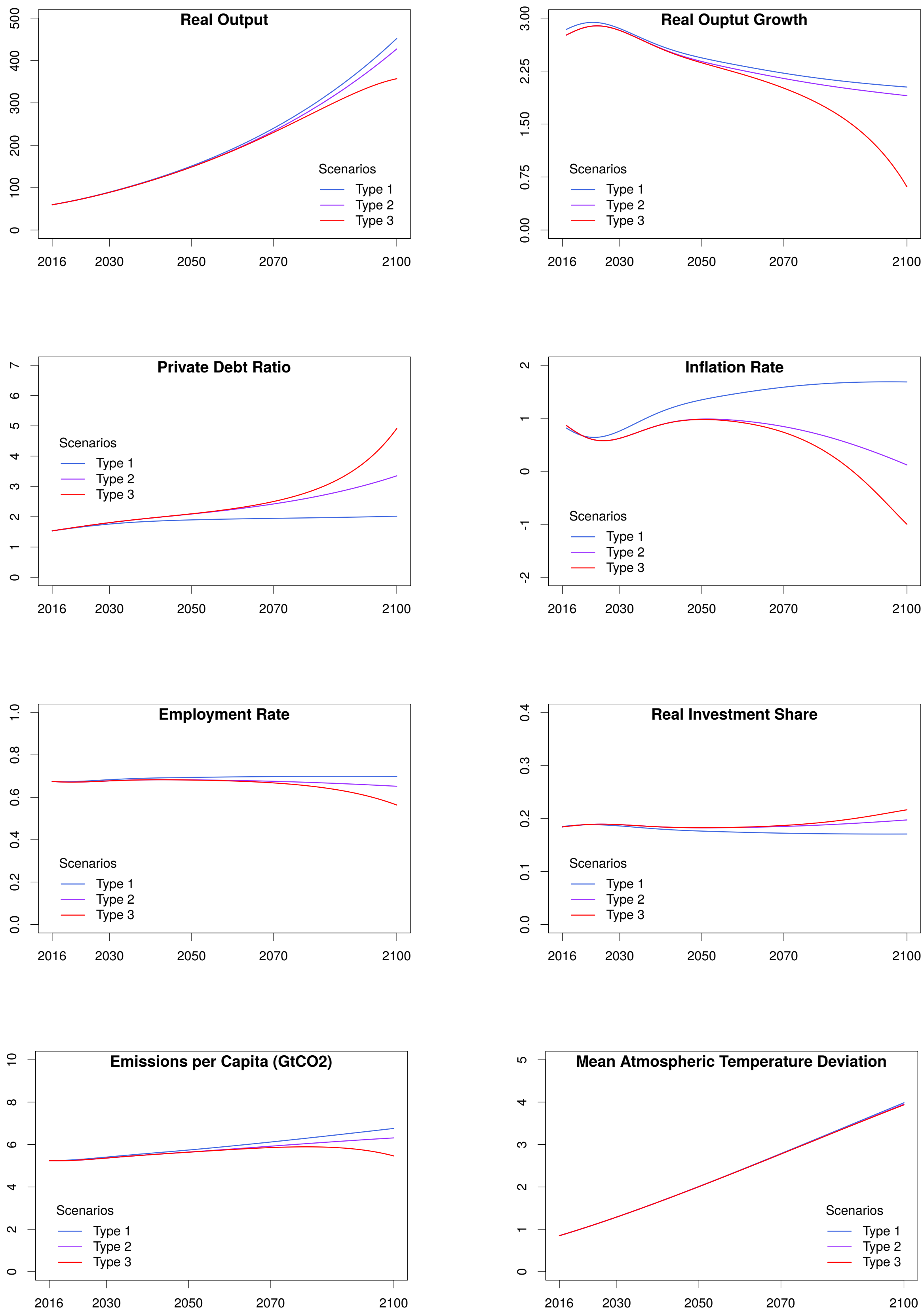

Figure 6: Macroeconomic trajectories without proactive public policies.

At first glance, one observes that, as expected, global warming systematically penalizes output, since in our three last scenarios the compound annual real growth rate between 2010 and 2100 remains below the Baseline scenario level by at least 2.50%. Moreover, climate change increases potential financial instability through a higher private debt ratio. Next, a clear-cut distinction emerges between a situation where damages affect only output (Type 1 scenario) and the case where capital is affected as well (Type 2 and 3 scenarios). In fact, a temperature-dependent depreciation rate of capital (Type 2) induces a sharper slowdown of output as the temperature rises and, more importantly, a debt-to-GDP ratio that is significantly higher from 2.02 (Type 1) to 3.35 (Type 2) in 2100. As for the Type 3 scenario, this leads to a global breakdown in the next century, as illustrated by Figure 7 *infra*.

5.4.1 Mild impact of climate change (Type 1 scenario)

Let us first compare the Type 1 scenario (where climate damages only slightly alter the economic trajectory) with the baseline scenario. In Type 1 scenario (in light blue), around the turn of the next century, high CO_2-e emissions (up to approximately 6.8 $GtCO_2$-e per capita in 2100) feed a significant temperature increase (approximately $+4°C$ in 2100 in the atmospheric layer), which in turn substantially aggravates damages to production. These losses induce a rise of the debt ratio, driving the world economy far from its long-run stationary point. This illustrates the logic highlighted in Section 4: the increase in environmental damages, $\dot{\mathbf{D}}^{\mathrm{Y}}$, is at its highest until the energy shift is completed.[34] At the same time, as the world population is plateauing, demography no longer contributes to output growth, which is driven solely by α, labor-augmenting technological progress. However, being penalized by global warming, output remains below its long-run potential: $g < \alpha$. As a result, the employment rate, λ, further declines. Indeed, $L = Y/(a(1-\mathbf{D}^{\mathrm{Y}})(1-A))$, so that

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{L}}{L} - \frac{\dot{N}}{N} - = g + \frac{\dot{\mathbf{D}}^{\mathrm{Y}}}{(1-\mathbf{D}^{\mathrm{Y}})} + \frac{\dot{A}}{(1-A)} - \alpha - \frac{\dot{N}}{N}.$$

Since

$$g = \frac{\kappa(\pi)}{\nu}(1-\mathbf{D}^{\mathrm{Y}})(1-A) - \delta - \frac{\dot{\mathbf{D}}^{\mathrm{Y}}}{(1-\mathbf{D}^{\mathrm{Y}})} - \frac{\dot{A}}{(1-A)},$$

we obtain

$$\frac{\dot{L}}{L} = \frac{\kappa(\pi)}{\nu}(1-\mathbf{D}^{\mathrm{Y}})(1-A) - \delta - \alpha \leq 0.$$

Meanwhile, the wage share, ω, decreases as a consequence of three forces conspiring together:

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} + \frac{\dot{L}}{L} - i - g.$$

The decline in λ (with respect to its long-run stationary value) in fact reduces wages via the short-run Phillips curve, and $\dot{\lambda}/\lambda < 0$, while $i, g > 0$. This narrative confirms the rather unrealistic feature of the climate-economy interaction modeling on which it is based. As we shall now see, the picture changes dramatically as soon as damages are allocated between output and capital.

5.4.2 Severe impact of climate change (Type 2 and 3 scenarios)

All but the Baseline and Type 1 scenarios exhibit a collapse of real output before the end of the twenty-second century, while the NAIRU falls to zero and the debt ratio explodes.[35] Type 3 scenario even suggests that degrowth (by disaster, not design) might start already at the turn of the next century, eventually leading to a collapse, as illustrated by the phase diagram in the (ω, λ, d) space of Figure 7 (the initial condition being located on the right side of the Figure).

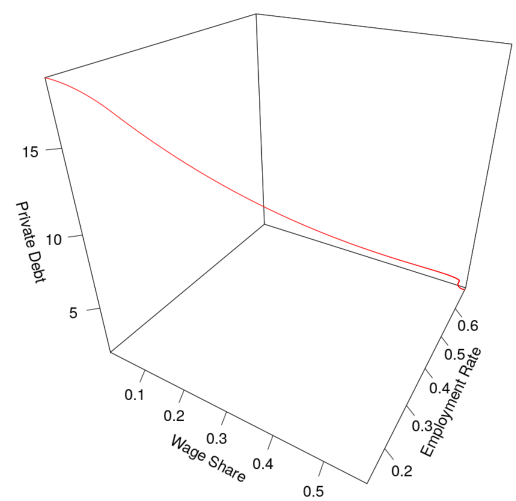

Figure 7: Phase diagram for the Type 3 scenario over the period 2016–2130.

Remember from Section 4 that global warming significantly shrinks the "good" basin of attraction. Our simulations therefore show that, at least within the conditions embodied in our last two scenarios, today's world economy lies outside the "good" basin of attraction shrunk by global warming. This raises an important question of public policy: which tools will enable public authorities to drive the world economy back into the "good" basin of attraction?

In Type 2 and 3 scenarios, climate disturbance now inflicts direct damages on capital, permanently reducing potential output. As a result, the world productive sector is

[34] Remember that in the Nordhaus scenario environmental damages only affect output.

[35] Of course, introducing default would make it possible to get rid of the unrealistic feature of unbounded private debts. This will be done in a subsequent paper.

forced to leverage in order to finance investment, which leads to a rise of the debt ratio. Moreover, real output is penalized, ultimately generating disruptive effects on the labor market, while "forced" degrowth occurs. It is worth mentioning that the type of degrowth (by disaster, not by design) just alluded to is characteristic of Fisherian debt-deflation. Indeed, as discussed in Section 4, global warming reduces the employment rate and the wage share while private debt skyrockets and negative inflation rates prevail, as observed in Fig. 1.

Notice also that all these scenarios are accompanied by a temperature deviation in 2100 higher than +3°C, hence far above the +2°C target of the Paris Agreement. Would a proactive emission-reduction rate path that manages to meet the +2°C imperative enable the world economy to avoid a downfall? And can we find some (possibly minimal) trajectory for a carbon price that would provide the right incentives to speed up the energy shift so as to achieve the +2°C target?

Scenario	**Baseline**	**Type 1**	**Type 2**	**Type 3**
CAGR of real GDP w.r.t. 2010-2100	2.50%	2.44%	2.37%	2.15%
Private debt ratio in 2100	1.81	2.02	3.35	4.91
CO_2-e emissions per capita in 2050	-	5.74 t CO_2-e	5.65 t CO_2-e	5.64 t CO_2-e
Temperature change in 2100	-	+3.99°C	+3.95°C	+3.94°C
CO_2-e concentration in 2100	-	819 ppm	804 ppm	792 ppm

Table 3: Key values of the world economy.

6 Target achievements

Given the exogenous backstop technology (borrowed from [Nordhaus, 2017]), a sufficiently fast-growing carbon price trajectory makes it *a priori* easier to implement a more intensive emission-reduction rate path, since it then becomes more costly to continue emitting greenhouse gases. In this section, we assess the achievability of various policy objectives through this simple mechanism.[36] Over the last decades, two main targets have been discussed on the international scene: +2°C and +1.5°C. The +2°C target has been advocated by the IPCC since 2003. First proposed by the European Union, this objective was discussed in Bali (2007) before being mentioned in Copenhagen (2009), finally adopted in Cancun (2010), and reaffirmed in Paris (2015). The objective of limiting global warming to +1.5°C was also mentioned as being desirable by the Paris Agreement (Art. 2). Before turning to the computation of a set of target achievements, we first analyzed the implications of using the calibration of [Nordhaus, 2017] for the climate module.

6.1 Which targets can be reached?

Our conclusion on the +2°C target is clear[37], though using an entirely different economic model: without capture and storage of carbon, it will be almost impossible to achieve this goal unless, by an extraordinary stroke of luck, climate sensitivity turned out to be equal to its floor value (1.5) . To have a better understanding of this impossibility result, let us look more closely at the temperature anomaly if zero net emissions are achieved prior to 2018.

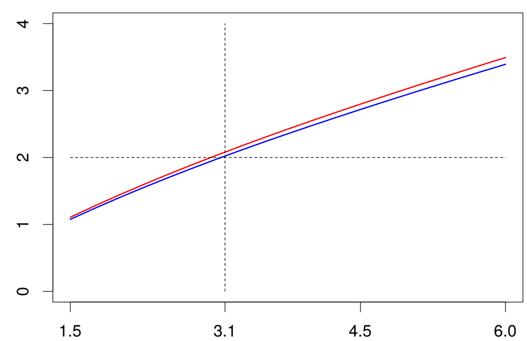

Figure 8: Temperature increase in 2100 as a function of climate sensitivity whenever zero net emission is reached in 2016 (blue line) or 2018 (red line)

Figure 8 yields the value of climate sensitivity (introduced in subsection 3.2 *supra*) on the x-axis and, on the y-axis, the temperature anomaly whenever carbon neutrality has been achieved (i) at the start of the simulation, i.e., in 2016, in blue; and (ii) in 2018, while 2016 and 2017 have a standard profile of emissions, in red.[38] As expected, whenever climate sensitivity increases, the temperature anomaly accelerates. For a climate sensitivity of 3.1,[39] the blue (resp. red) curve reaches a temperature anomaly of 2.022 (resp.

[36] See the Stern-Stiglitz report [Co-chaired by Stiglitz and Stern, 2017] for an assessment of a corridor of needed carbon prices, and why carbon pricing is anyway not sufficient: regulations and structural reforms are needed as well.

[37] And amply confirms the conclusion reached by [Nordhaus, 2017].

[38] For the sake of precision, the heat capacity of the atmosphere C is updated according to the climate sensitivity in order to account for the changes in the TCR when the ECS changes as in [Nordhaus and Sztorc, 2013]. Furthermore, we preclude carbon storage in our analysis.

[39] 3.1 is approximately the mean value of the distribution $\log - \mathcal{N}(\mu = 1.10704, \sigma = 0.264)$ reported in [Gillingham et al., 2015].

2.081) in 2100.

In light of these results, we now examine the feasibility of less demanding temperature objectives under the assumption of a climate sensitivity (ECS) equal to 3.1.

6.2 Which shape for the carbon price?

In our framework, mitigation policies are driven by carbon pricing. The shape of the latter influences significantly both the temperature trajectories and the economic outcome of the model. So far, however, only the exponential shape introduced by Eq. 33 has been used. To allow for more flexibility, we now consider more general sigmoid carbon price trajectories:[40]

$$\frac{\dot{p_C}}{p_C} = \beta_{pC}\left(1 - \gamma_{pC}\frac{p_C}{p_{BS}}\right), \tag{38}$$

with, $\beta_{pC} > 0$, and $1 \geq \gamma_{pC} \geq -1$. Note that, when $\gamma_{pC} = 0$, Eq. 38 reduces to the exponential case of Eq. 33.

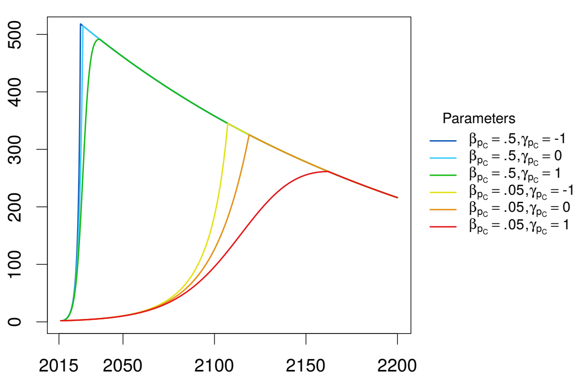

Figure 9: Examples of carbon price paths included in our calibration set

Figure 9 shows the four types of shapes allowed by Eq. 38 for the parameters $(\beta, \gamma) \in \{.05, .5\} \times \{-1, 1\}$, together with two exponential cases. This allows us to envisage different stories that go from very fast energy shifts in the spirit of Nicholas Stern—with a carbon pricing even more convex than in the exponential case—to slower transitions, closer to the spirit of William Nordhaus. In all cases, whenever the carbon price hits the price of the backstop technology, the former equals the latter and zero net emission prevails—carbon pricing then decreases almost linearly. Which targets can be reached using this carbon pricing?

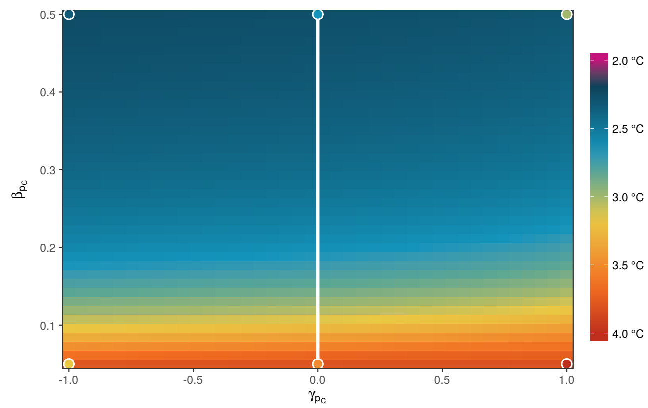

Figure 10: Heatmap depending on the carbon price path in the Type 3 scenario with ECS=3.1.

Figure 10 plots the heatmap of the temperature increase depending on the two carbon pricing parameters $(\gamma_{pC}, \beta_{pC})$ that vary within the set $[-1; 1] \times [0, 0.5]$. The model is simulated for each combination and the value found for the temperature anomaly in 2100 is represented by a color (going from +2°C in purple, up to +4°C in red). Each dot within the heatmap represents a carbon price trajectory as displayed in Figure 9, the color being identical in both figures. Finally, the white line ($\gamma_{pC} = 0$) represents the exponential case. Echoing subsection 6.1, the purple color is absent from Figure 10. This suggests that, even with a very high carbon price, +2°C can no longer be reached unless ECS turns out to be extremely low or CCS technologies are deployed on an industrial scale in the coming decades. However, very high carbon prices suggest that an increase in temperature of between +2.1°C and +2.5°C can still be reached, although Figure 9 shows that this requires a very steep and early carbon pricing. As a consequence, a target achievement of +3°C seems to us to be a more realistic option. Finally, it is worth mentioning that the γ_{pC} parameter has little impact on the final outcome in terms of temperature increase, suggesting that all that matters is β_{pC}, i.e., the speed at which the energy shift takes place.

Building on these results, we now analyze what seem to be more realistic achievement goals—namely a +2.5°C and +3°C limitation of global warming[41]—and discuss their consequences in terms of financial stability.

6.3 New achievement goals: +2.5°C and +3°C in 2100

Figure 11 plots our results within the Type 1 scenario. In this setting, a carbon price of, e.g., 2010 US$ 2 in 2016, $5 in 2020, $108 in 2030, and $433 in 2040 per GtCO2 would

[40] To avoid technicalities involved in variational calculus, we limit ourselves to this family of generalized sigmoid carbon price paths, the derivation of more general carbon price paths being left for further research.
[41] To the best of our knowledge, neither +2.5°C nor +3°C have ever been invoked in international circles.

be required to achieve a +2.5°C objective in 2100. For a +3°C goal, the figures would be 2010 US$ 2 in 2016, $3.1 in 2020, $14 in 2030, $ 57.2 in 2040, $182 in 2050 and $338.8 in 2060 per GtCO2.[42]

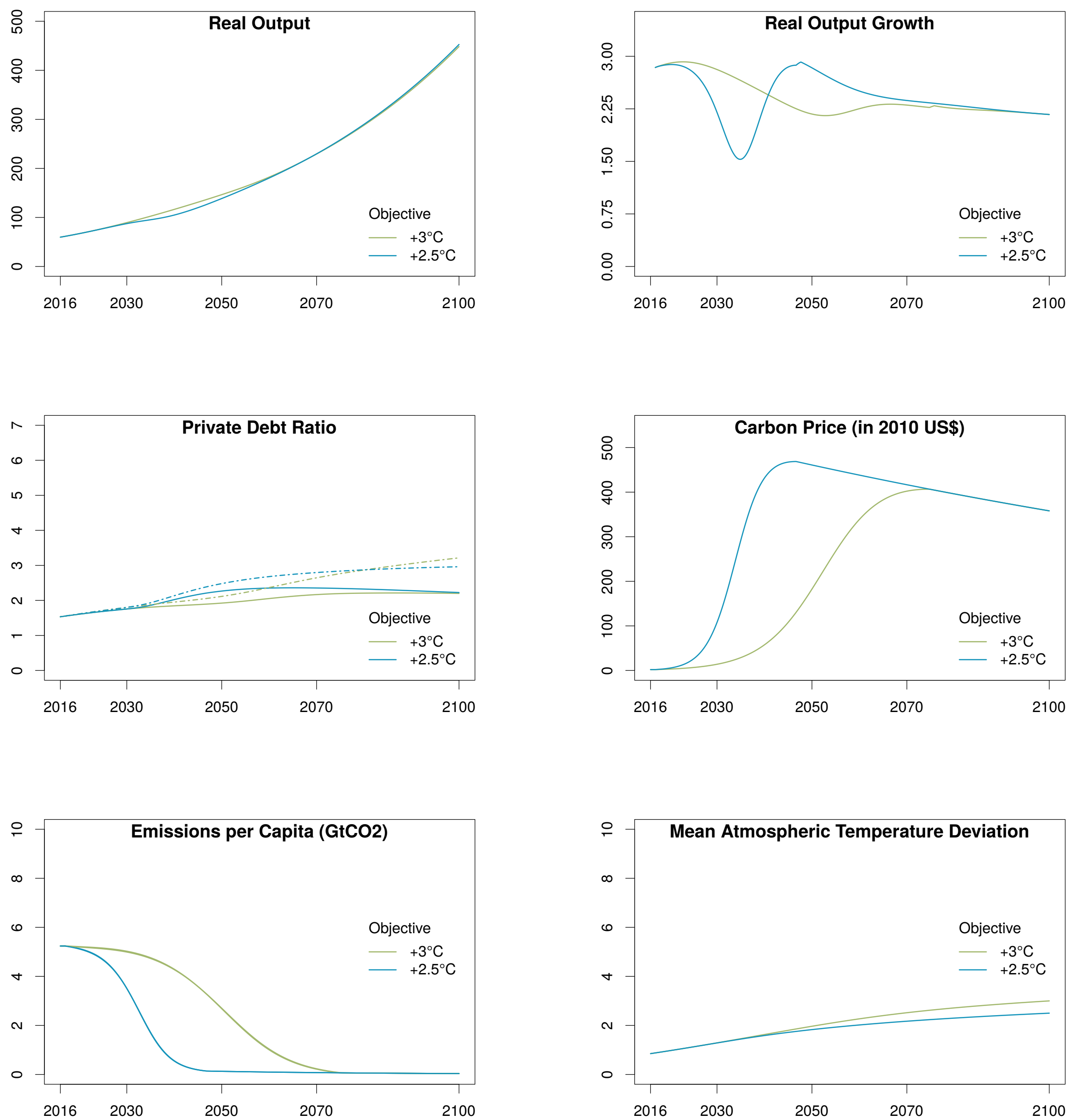

Figure 11: Trajectories for the Type 1 scenario compatible with +2.5°C and +3°C (in dashed curves, the trajectories for the private debt ratio in the Type 3 scenario).

All the trajectories in light blue (resp. in green) exhibit a mean atmospheric temperature of +2.5°C (resp. +3°C) at the end of this century, courtesy of an energy shift achieved around 2045 (resp. 2070). This means that carbon neutrality needs to be reached at the world level as early as 2045 (resp. 2070) if the +2.5°C (resp. +3°C) is to be met.[43] For the sake of simplicity, Figure 11 only plots the Type 1 scenario since the other scenarios show almost identical patterns—with the exception of the debt-to-output ratio, which is displayed in dashed curves for the Type 3 scenario.

Comparing the +2.5°C trajectories with their +3°C

[42] For this thought exercise, we assume $\gamma_{p_C} = 1$, and look for the unique value of, β_{p_C}, compatible with the achievement of the objective under scrutiny. Figure 10 shows that the parameter, β_{p_C}, is the most discriminant.

[43] This is in line, e.g., with the Deep Decarbonization Pathway initiative (2015) [IDDRI, 2015].

counterparts, the main difference is that the real GDP in 2100 is 1% higher as the temperature is lower. This means that, by the end of the century, the extra abatement cost of reducing the temperature anomaly by +0.5°C will pay off as it makes damages less harmful to the economy. Moreover, Figure 11 shows that the real output growth rate drops whenever the emission-reduction effort is made. This phenomenon is amplified as the transition date intervenes earlier. Furthermore, the economic real growth rate goes back to its long-term steady-state value—i.e., declining as the demographic transition occurs.

Turning to the private nonfinancial corporate debt ratio, the effect of the abatement cost can also be seen on the debt-to-output ratio: in 2050 (resp. 2100) the +2.5°C (resp. +3°C) trajectory exhibits a debt-to-output ratio of 2.27 (resp. 2.22) while its +3°C counterpart is 1.93 (resp. 2.2). A faster energy-shift clearly translates into a greater potential source of financial instability.[44]

Last, when running these simulations within a more realistic set-up—the Type 3 scenario—, where environmental damages are also factored into the depreciation rate of capital, we see that the world economy is facing a run-up of debt. In 2100, our simulations show a 3.1 (resp. 3.57) debt-to-GDP ratio for the +2.5°C trajectory (resp. +3°C). At the same time, the market value of capital is only 2.65 (resp. 2.74). These values raise a red flag: whenever nominal debt becomes higher than money capital (which usually serves as collateral), a cascade of defaults is to be expected, along the storyline analyzed (in a static framework) by [Fostel and Geanakoplos, 2015].

7 Conclusion

By combining financial and environmental features, the stock-flow consistent macroeconomic model introduced in this paper allows us to examine the conditions for economic growth, or else possible (forced) degrowth, depending on the damages induced by global warming and the carbon price path. To our knowledge, this is the first dynamic model calibrated at the world level that enables both environmental and financial risks to be assessed within a framework of endogenous monetary business cycles.

Our main findings are as follows. First, the $+2$°C target seems already out of reach unless the climate sensitivity turns out to be very low (1.5) or we develop negative emission technologies at the world scale in the upcoming decades.

Second, our simulations shed new light on the interplay between financial (level of private debt) and climate instabilities. Indeed, both reinforce each other and may ultimately lead to an unintended planet-wide economic degrowth at the end of this century (cf. our Type 3 scenario). However, redistributing wealth so as to increase the wage share, fostering employment, and reducing the private debt-to-output ratio, however, would make it easier for today's world economy to move towards to the "good" basin of attraction, i.e., to find a growth path that will ultimately circumvent a breakdown. In other words, coping with collapse on a hotter planet means, among other things, private deleveraging, income distribution in favor of workers, and a high employment rate. To our knowledge, this is the first time these channels have been shown to facilitate adaptation to climate change.

Third, the implementation of an adequate policy of emission-reduction through the deployment of a carbon price trajectory enables long-term prosperity to be restored, at least whenever climate sensitivity is 3.1°C. According to the simulations performed in this paper, however, the binding carbon price trajectory must be such that the energy shift be completed, or zero net emission be reached as early as 2040 and, in any case, *before 2070* in order to maintain global warming bellow a +3°C target. For instance, within our framework, a carbon price of, e.g., 2010 US$ 2 t/CO_2-e_2 in 2016, $108 in 2030 and $433 in 2040 is compatible with the achievement of the +2.5°C objective.

These results *a posteriori* justify our choice not to follow a standard cost-benefit analysis to assess the impact of climate-driven externalities. Indeed, the latter approach inevitably ends up with the issue of calibrating the "right" discount rate. While substantial efforts have been devoted to assessing whether a high or low, and sometimes a time-varying discount rate should be considered,[45] none of this literature, to the best of our knowledge, has ever considered a negative rate.[46] Yet, this possibility should be seriously envisaged. Not only because of the pervasive negative real interest rates observed nowadays on international markets, but also, as shown in this paper, because a world breakdown might be the prospect that markets should start considering as possible from now on. If the next generation is going to be less wealthy than we are today, then a US dollar today should be worth less than the same dollar in a couple of decades.

Finally, this paper calls for a number of extensions. We mention only a few of them for the sake of brevity: can an appropriate tax policy implement the type of income redistribution that would favor adaptation to climate disorder? For simplicity, we assumed strict complementarity between capital and labor. Would some (more realistic) subsitutability ease the task of the production sector when compensating losses inflicted by global warming? Or would it favor unemployment, and hence, following the findings of this pa-

[44] This confirms the warning strongly expressed by Governor Mark Carney in September 2015 [Carney, 2015].

[45] see, e.g., [Sterner and Persson, 2008]

[46] Except for [Ekeland, 2015], p.49, who introduces an "ecological interest" rate. According to Ekeland, consumption goods (available in large quantities) and natural resources (available in limited quantities) should be valued using two different interest rates. While the first one can be set by the market, the second one should be possibly negative due to its finiteness.

per, make it harder to circumvent a cave-in? What if, instead of behaving myopically, economic actors were to share (possibly wrong) expectations about the near future? Finally, throughout the paper, we have taken the demographic trend as exogenous: would curbing the demographic curve—say, by means of some systematically implemented family planning policy—enable us to reach the 2°C challenge more easily? All of this obviously raises crucial challenges for future research.

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Appendices

A Stability analysis of the desirable steady state

Turning to the local asymptotic stability analysis of the long-run "good" equilibrium, the Jacobian matrix of the dynamic system reads

$$M(\omega_{eq}, \lambda_{eq}, d_{eq}, N_{eq}) = \begin{bmatrix} 0 & M_{12} & 0 & 0 \\ -M_{21} & 0 & -rM_{21} & M_{24} \\ M_{31} & 0 & M_{33} & 0 \\ 0 & 0 & 0 & M_{44} \end{bmatrix},$$

where the entries $M_{ij}, (i,j) \in [\![1;3]\!]$ are given by:

$$\begin{aligned} M_{12} &:= \omega_{eq}\phi'(\lambda_{eq}) > 0, \\ M_{21} &:= \frac{\lambda_{eq}}{\nu}\kappa^{'}(pi_{eq})(1-\mathbf{D}^{\mathrm{Y}}{}_{eq}) > 0, \\ M_{24} &:= \lambda_{eq}\frac{q}{P^N}, \\ M_{31} &:= \left(\frac{d_{eq}(1-\mathbf{D}^{\mathrm{Y}}{}_{eq})}{\nu} - 1\right)\kappa^{'}(\pi_{eq}, \mathbf{D}^{\mathrm{Y}}{}_{eq}) - \Delta'(\pi_{eq}) + 1, \\ M_{33} &:= rM_{31} - \alpha, \\ M_{44} &:= -\frac{q}{P^N}. \end{aligned}$$

Then, the characteristic polynomial $\chi_M(\cdot)$ of the Jacobian matrix at the "good" equilibrium writes

$$\begin{aligned} \chi_M(\epsilon) &= \left(\epsilon + \frac{q}{P^N}\right)\big[\epsilon^3 + (\alpha - rM_{31})\epsilon^2 + ... \\ &\quad ... + M_{12}M_{21}\epsilon + g_{Y_{eq}}M_{12}M_{21}\big]. \end{aligned}$$

The first root, $\epsilon = -\frac{q}{P^N}$, of the polynomial $\chi_M(\cdot)$ being obviously negative, the stability of the "good" equilibrium is given by the sign of the root of its factored polynomial of degree 3.[47] According to the Routh-Hurwitz criterion, a necessary and sufficient condition for the root of this polynomial to have a negative and non-null real part is

1. $\alpha > rM_{31}$,
2. $(\alpha - rM_{31})M_{12}M_{21} > \alpha M_{12}M_{21}$, which is equivalent to $rM_{31} < 0$ since M_{12} and M_{21} are positive, and α non-negative.

[47] The latter is similar to the characteristic polynomial found by [Grasselli and Lima, 2012].

B Calibration of the Model

Symbol	Description	Value	Remarks and sources
C	Heat capacity of the atmosphere, biosphere and upper ocean	1/.098	DICE model, [Nordhaus, 2017], adjusted for a continuous framework
C_0	Heat capacity of the deeper ocean	3.52	DICE model, [Nordhaus, 2017], adjusted for a continuous framework
$C_{AT_{pind}}$	CO_2-e preindustrial concentration in the atmosphere layer	588 Gt C	DICE model, [Nordhaus, 2017]
$C_{UP_{pind}}$	CO_2-e preindustrial concentration in the biosphere and upper ocean layer	360 Gt C	DICE model, [Nordhaus, 2017]
$C_{LO_{pind}}$	CO_2-e preindustrial concentration in the deeper ocean layer	1,720 Gt C	DICE model, [Nordhaus, 2017]
div_0	Constant of the dividend function, $\Delta(\cdot)$	-0.078	Empirically calibrated, macroeconomic database, more details available upon request
div_π	Slope of the dividend function, $\Delta(\cdot)$	.473	Empirically calibrated, macroeconomic database, more details available upon request
$div_{\min}$	Minimum of the dividend function,$\Delta(\cdot)$	0	Selected among a range of reasonable values
$div_{\max}$	Maximum of the dividend function, $\Delta(\cdot)$	0.3	Selected among a range of reasonable values
$F_{2\times \text{CO}_2}$	Change in the radiative forcing resulting from a doubling of CO_2-e concentration w.r.t. to the pre-industrial period	3.681 W/m^2	DICE model, [Nordhaus, 2017]
F_{exo}^{start}	Initial value of the exogenous radiative forcing	0.7 W/m^2	DICE model, [Nordhaus, 2017]
F_{exo}^{end}	Final value of the exogenous radiative forcing	0.7 W/m^2 (after 2100)	DICE model, [Nordhaus, 2017]
f_K	Fraction of environmental damage allocated to the stock of capital	1/3	[Dietz and Stern, 2015] and [Moyer et al., 2014]
P^N	Upper limit of the workforce dynamics in billions	7.056	Empirically calibrated, macroeconomic database, more details available upon request
P_G^N	Upper limit of the total population dynamics in billions	12	Empirically calibrated, macroeconomic database, more details available upon request
q	Speed of growth of the workforce dynamics	0.0305	Empirically calibrated, macroeconomic database, more details available upon request
q_G	Speed of growth of the total population dynamics	0.027	Empirically calibrated, macroeconomic database, more details available upon request
r	Short-term interest rate of the economy	0.02	Selected among a range of reasonable values
S	Equilibrium climate sensitivity	3.1 °C	DICE model, [Nordhaus, 2017]
T_{preind}	Preindustrial temperature	13.74°C	NASA (2016)[NASA, 2016]
α	Constant growth rate of labor productivity	0.02	Selected among a range of reasonable values
δ	Depreciation rate of capital	0.04	[Inklaar and Timmer, 2013]
$\delta_{E_{Land}}$	Growth rate of land use change CO_2-e emissions	-0.022	DICE model, [Nordhaus, 2017], adjusted for a continuous framework
δ_{g_σ}	Variation rate of the growth of emission intensity	- 0.001	DICE model, [Nordhaus, 2017], adjusted for a continuous framework
$\delta_{p_{BS}}$	Exogenous growth rate of the back-stop technology price	- 0.005	DICE model, [Nordhaus, 2017], adjusted for a continuous framework
ζ_3	Damage function parameter	6.754	DICE model, [Nordhaus, 2017]
η	Relaxation parameter of the inflation	0.5	Selected among a range of reasonable values
θ	Parameter of the abatement cost function	2.6	DICE model, [Nordhaus, 2017]
κ_0	Constant of the investment function, $\kappa(.)$	0.0318	Empirically estimated, macroeconomic database, more details available upon request
κ_1	Slope of the investment function, $\kappa(.)$	0.575	Empirically estimated, macroeconomic database, more details available upon request
κ_{min}	Minimum of the investment function, $\kappa(.)$	0	Selected among a range of reasonable values
κ_{max}	Maximum of the investment function, $\kappa(.)$	0	Selected among a range of reasonable values
μ	Mark-up of prices over the average cost	1.3	Selected among a range of reasonable values
ν	Constant capital-to-output ratio	2.7	[Inklaar and Timmer, 2013]
π_1	Damage function parameter	0 /°C	DICE model, [Nordhaus, 2017], adjusted for a continuous framework
π_2	Damage function parameter	0.00236/°C^2	DICE model, [Nordhaus, 2017]
π_3	Damage function parameter in the Weitzman case	0.00000507/°C$^{\zeta_3}$	[Weitzman, 2011] and [Dietz and Stern, 2015]
ϕ_0	Constant of short-term Phillips curve, $\phi(.)$	-.292	Empirically estimated, macroeconomic database, more details available upon request
ϕ_1	Slope of the short-term Phillips curve, $\phi(.)$	.469	Empirically estimated, macroeconomic database, more details available upon request
Φ_{12}	Transfer coefficient for carbon from the atmosphere to the upper ocean/biosphere	0.024	DICE model, [Nordhaus, 2017], adjusted for a continuous framework
Φ_{23}	Transfer coefficient for carbon from the upper ocean/biosphere to the lower ocean	0.001	DICE model, [Nordhaus, 2017], adjusted for a continuous framework

The mentioned macroeconomic database gathers data from the World Bank, Penn University, the U.S. Bureau of Economic Analysis, and the United Nations.

C Initial values of the Model

Symbol	Description	Value	Remarks/sources
CO_2^{AT}	CO_2-e concentration in the atmosphere layer	851 Gt C	DICE model, [Nordhaus, 2017]
CO_2^{UP}	CO_2-e concentration in the biosphere and upper ocean layer	460 Gt C	DICE model, [Nordhaus, 2017]
CO_2^{LO}	CO_2-e concentration in the deeper ocean layer	1,740 Gt C	DICE model, [Nordhaus, 2017]
d	Private debt ratio of the economy	1.53	Empirically calibrated, macroeconomic database
E_{ind}	Industrial CO_2-e emissions	35.85 Gt CO_2-e	DICE model, [Nordhaus, 2017]
E_{land}	Exogenous land use change CO_2-e emissions	2.6 Gt CO_2-e	DICE model, [Nordhaus, 2017]
F_{exo}	Exogenous radiative forcing	0.5 W/m^2	DICE model, [Nordhaus, 2017]
g_σ	Growth rate of the emission intensity of the economy	- 0.0152	DICE model, [Nordhaus, 2017]
p	Composite good price level	1	Normalization constant
p_{BS}	Backstop price level	547.22	DICE model, [Nordhaus, 2017], compound 1-year ahead
n	Emissions reduction rate	0.03	DICE model, [Nordhaus, 2017]
N	Workforce of the economy in billions	4.83	Empirically calibrated, macroeconomic database
NG	Total population in billions	7.35	Empirically calibrated, macroeconomic database
T	Temperature in the atmosphere, biosphere and upper ocean layer	0.85 °C	DICE model, [Nordhaus, 2017]
T_0	Temperature in the deeper ocean layer	0.0068 °C	DICE model, [Nordhaus, 2017]
Y	Gross domestic product (at factor prices) in trillions USD	59.74	Empirically calibrated, macroeconomic database
λ	Employment rate of the economy	0.675	Empirically calibrated, macroeconomic database
ω	Wage share of the economy	0.578	Empirically calibrated, macroeconomic database

The mentioned macroeconomic database gathers data from the World Bank, Penn University, the U.S. Bureau of Economic Analysis, and the United Nations.

D Estimation of behavioral equations

Three behavioral aggregate functions are empirically estimated, the investment function, $\kappa(\cdot)$, the short-term Phillips curve, $\Phi(\cdot)$, and the dividend function, $\Delta(\cdot)$. This Appendix outlines the methodology followed after introducing the data.

D.1 The data

The data collection gathers four databases for various time-series:

- World Bank: GDP (current 2010 US$); Employment-to-population ratio, 15+, total (%) (modeled ILO estimate); Population ages 65 and above (% of total); Population, total and gross fixed capital formation (current US$);
- Penn World Table: Share of Labor Compensation in GDP at National Prices;
- BIS: Private Debt Non-Financial sector (All sectors, Market value, Percentage of GDP, and Adjusted for breaks);
- FRED St. Louis: Corporate Profits after tax with IVA and CCAdj: Net Dividends, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate.

To cope with missing data, as time-series are not available within the same time frame, we selected countries with data available from 1991 to 2014. The list of selected countries includes: Argentina, Australia, Austria, Belgium, Canada, Chile, China, Denmark, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Malaysia, Mexico, Netherlands, Norway, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, Thailand, Turkey, the United Kingdom, and the United States.

For simplicity, we assumed a fixed interest rate of 3%. To wrap-up, we were able to retrieve time-series for: the wage growth, the employment rate, the investment-to-GDP ratio, the retained profit rate, and the dividend rate.

D.2 The methodology

As multinational corporations have become more pervasive, major currency changes have been made (e.g., euro zone), and financial instability has frequently had an impact over the 2000s, it is difficult to find consistency between data in the 1990s and the 2000s. For these reasons, we estimated the coefficients of the three behavioral function—the investment function, $\kappa(\cdot)$, the short-term Phillips curve, $\Phi(\cdot)$, and the dividend function, $\Delta(\cdot)$—by minimizing the mean square error for data in the aftermath of the burst of the dot-com bubble (i.e. from 2001 on). The estimated empirical forms are linear. The values found are the ones reported in Appendix B. More details will be available upon request.