Forward Guidance without Common Knowledge

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Forward Guidance: Context or Pretext?

- How does the economy respond to news about the future?
  - e.g., news about future interest rates or government spending

- Key mechanisms:
  - forward-looking expectations (e.g., of inflation and income)
  - general-equilibrium effects (Keynesian multiplier, π-y feedback)

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- Standard: RE with CK
- This paper: RE without CK

Main Insight and Applications

- Removing CK reduces
  - responsiveness of forward-looking expectations
  - potency of GE effects (Keynesian multipliers etc)

- Effects increase with horizon
  - it is as if agents apply extra discounting on future outcomes

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- Effects increase with horizon
  - it is as if agents apply extra discounting on future outcomes
- Application to ZLB context
  - arrest response of AD to news about interest rates
  - arrest response of inflation to news about marginal costs
  - lessen forward guidance puzzle
  - offer rationale for the front-loading of fiscal stimuli
  - ...

Roadmap

1. Recast IS and NKPC as Dynamic Beauty Contests
2. Show GE Attenuation and Horizon Effects
3. Application to Forward Guidance and Fiscal Stimuli
4. Comparison to Related Work that Drops RE

Mapping the IS and the NKPC to Dynamic Beauty Contests

Framework

- Starting point: textbook NK model
- Main departure: remove CK of innovations in fundamentals/policy
- Auxiliary: enough "noise" to prevent revelation through prices
  - variant with similar results: rational inattention
- Key friction: uncertainty about how others will respond
  - uncertainty about future inflation and income
  - not uncertainty about the fundamentals/policy per se
  - to understand how it matters $\rightarrow$ IS and NKPC as beauty contests

The Euler/IS Curve with Common Knowledge

$$c_t = -E_t[r_{t+1}] + E_t[c_{t+1}]$$

- Key implication: $c = f(\text{expected path of } r)$
  - implication robust to borrowing constraints (e.g., McKay et al)
  - even though the aggregate Euler equation itself is different

The Euler/IS Curve without Common Knowledge

$$c_t = -\left\{\sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[r_{t+k}]\right\} + (1-\beta)\left\{\sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t\left[c_{t+k}\right]\right\}$$

- Defines a dynamic beauty contest among the consumers
- Key implication: $c \neq$ f(expected path of r). Instead, HOB matter.

The NK Philips Curve with Common Knowledge

$$\pi_t = mc_t + \beta E_t [\pi_{t+1}]$$

- Key implication: π = f (expected path of mc)

The NK Philips Curve without Common Knowledge

$$\pi_t = mc_t + \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \, \bar{E}_t^f \left[mc_{t+k} \right] \right\} + \frac{1-\theta}{\theta} \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \, \bar{E}_t^f \left[\pi_{t+k} \right] \right\}$$

- Defines a dynamic beauty contest among the firms
- Key implication: $\pi \neq$ f(expected path of mc). Instead, HOB matter

So Far, and What's Next

- So far: represent IS and NKPC as dynamic beauty contests
- What's next: the beauty of dynamic beauty contests!
  - consider a more abstract setting (nests other applications too)
  - develop broader insights
  - apply insights to context of interest
- Note: Higher Order Beliefs = a window to Rational Expectations

Attenuation and Horizon Effects in Dynamic Beauty Contests

An Abstract Dynamic Beauty Contest

- Consider models in which the following Euler-like condition holds:

$$a_{i,t} = \theta_t + \gamma E_{it}[a_{i,t+1}] + \alpha E_{it}[a_{t+1}]$$

  - θ_t = fundamental, a_{it} = individual outcome, a_t = aggregate outcome
  - $\gamma > 0$ parameterizes PE effects, $\alpha > 0$ parameterizes GE effects

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- Iterate over t and aggregate over $i \Rightarrow$ dynamic beauty contest

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- With CK $\Rightarrow$ representative-agent Euler

$$a_t = \theta_t + (\gamma + \alpha) E_t[a_{t+1}]$$

Question of Interest

- How does a_t responds to news about θ_{t+T} ?
  - c response to news about interest rates
  - π inflation response to news about marginal costs

- Formally:
  - hold θ_τ constant (say, at 0) for all $\tau \neq t + T$
  - treat θ_{t+T} as a random variable (Normally distributed with mean 0)
  - study $\phi_T \equiv$ projection coefficient of a_t on $\bar{E}_t[\theta_{t+T}]$

The Role of HOB

- By iterating, we can express a_t as a linear function of
  - 1st-order beliefs: $\bar{E}_t\left[\theta_{t+T}\right]$
  - 2nd-order beliefs: $\bar{E}_t\left[\bar{E}_\tau\left[\theta_{t+T}\right]\right] \;\; \forall \tau : t < \tau < t+T$
  - 3rd-order beliefs: $\bar{E}_t\left[\bar{E}_\tau\left[\bar{E}_{\tau'}\left[\theta_{t+T}\right]\right]\right] \;\; \forall \tau, \tau' : t < \tau < \tau' < t+T$
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- With CK, HOB collapse to FOB, the "usual" scenario applies, and

$$\phi_T^* = (\gamma + \alpha)^T$$

- Without CK, things are more tricky: ϕ_T hinges on
  1. how HOB co-move with $\bar{E}_t[\theta_{t+T}]$
  2. how HOB load in a_t

Two Basic Insights

1. HOB vary less than FOB
   - "unless I am 100% sure that you heard and paid attention to the news, I am likely to think that your beliefs moved less than mine"

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2. Longer horizons raise the relative importance of HOB
   - the distant future enters through multiple rounds of GE effects:

$$\theta_{t+T} \to a_{t+T} \to a_{t+T-1} \to ... \to a_{t+1} \to a_t$$

   - but this is akin to ascending the hierarchy of beliefs!
   - longer horizons therefore raise the load of HOB on outcomes

Results

1. Attenuation at any horizon
   - ϕ_T bounded between PE effect and CK counterpart:

$$\gamma^T < \phi_T < \phi_T^* = (\gamma + \alpha)^T$$

   - "CK maximizes GE effect"

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3. Attenuation effect grows without limit
   - $\phi_T / \phi_T^* \to 0$ as $T \to \infty$ even if noise is tiny*

Leading example

- Information structure:
  - each agent receives a private Gaussian signal about θ_{t+T} at t
  - no other info arrives up to $t+T$, at which point θ_{t+T} becomes known

- Implication: a simple exponential structure for HOB

$$\bar{E}_t^h[\theta_{t+T}] \;=\; \lambda^{h-1} \cdot \bar{E}_t[\theta_{t+T}]$$

where $\lambda \in (0, 1]$ is decreasing in the amount of noise

Leading Example

- Back to our question: How does a_t vary with $\bar{E}_t[\theta_{t+T}]$?
- Answer: Same as in a representative-agent model with

$$a_t = \theta_t + (\gamma + \lambda\alpha)E_t[a_{t+1}]$$

  - GE effect reduced from α to $\lambda\alpha$
  - as if myopia / extra discounting of future outcomes

Remarks and Take-Home Lessons

- Origins and interpretation of lack of CK
  - dispersed info as in Lucas, Grossman-Stigltiz, Morris-Shin, etc
  - bounded rationality in the form of "rational inattention" (Sims) and "costly contemplation" (Tirole)
  - key friction: uncertainty about responses of others
- Forget HOB, think Rational Expectations
  - the analyst has to think HOB, the agents inside the model do not!
  - we have merely "liberated" RE from the auxiliary CK restriction
- Take-home lessons
  - GE effects are less potent
  - economy may react as if agents were myopic
  - especially vis-a-vis news at more distant horizons

Going back to the NK model

- Demand block (IS):
  - attenuate GE feedback b/w c and y (Keynesian multiplier)
  - anchor income expectations
  - arrest response of c to news about future real rates
  - as if extra discounting in the Euler condition

- Supply block (NKPC):
  - attenuate GE feedback from future to current π
  - anchor inflation expectations
  - arrest response of π to news about future marginal costs
  - as if extra discounting in the NKPC

What's Next: Application to ZLB Context

- Caveat to applying preceding lessons:
  - GE feedback b/w demand (IS) and supply (NKPC)
  - joint endogeneity of real rates and real marginal cost
- Next: deal with this caveat
- Obtain lessons for forward guidance, fiscal stimuli, etc

Forward Guidance and Fiscal Stimuli

ZLB and Forward Guidance

- Let T index length of liquidity trap and horizon of FG
  - $t < T - 1$: ZLB binds and $R_t = 0$ for all
  - $t \geq T + \Delta$: "natural level" and $y_t = \pi_t = 0$
  - let $\Delta = 1$ for simplicity
- Forward guidance
  - policy announcement at $t = 0$ of likely R_T
  - modeled as $z = R_T + noise$
- Our twist: lack of CK about z
  - credibility = precision of z, or how much $\bar{E}_0[R_T]$ varies with z
  - we bypass this and focus on how y_0 varies with $\bar{E}_0[R_T]$
  - think of this as studying the response of spending and inflation relative to the response of the term structure of interest rates

Leading Example

- Information structure
  - initial private signal

$$x_i = z + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

  - ϵ_i can be interpreted as the product of rational inattention
  - limit with no endogenous learning (large markup and wage shocks)

- Degree of CK indexed by $\lambda \in (0, 1]$

$$\bar{\mathbb{E}}^h[R_T] = \lambda^{h-1}\bar{\mathbb{E}}^1[R_T]$$

  - consumers vs firms: λ_c vs λ_f
  - CK benchmark nested with $\lambda_c = \lambda_f = 1$

The Power of Forward Guidance

- *Question:* How does y_0 vary with $\bar{E}_0[R_T]$

- *Answer:* There exists a function ϕ such that

$$y_0 = -\phi\left(\lambda_c, \lambda_f; T\right) \cdot \bar{E}_0[R_T]$$

  - standard: ϕ^* increases with T and explodes as $T \to \infty$
  - here: ϕ vs ϕ^*

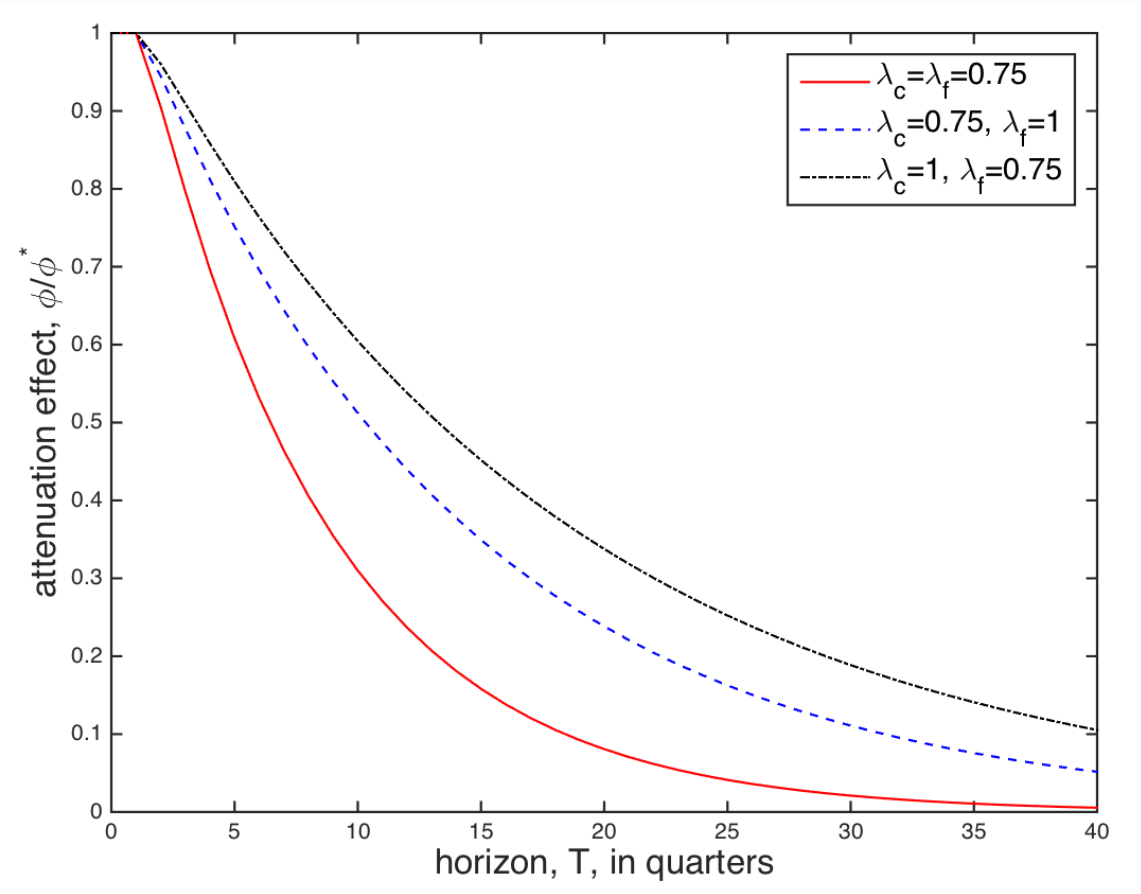
Main Results

- Attenuation for any horizon
  - three GE effects at work:
    (1) inside IS: income-spending feedback
    (2) inside NKPC: inflation-inflation feedback
    (3) across two blocks: inflation-spending feedback
  - all three attenuated; but most quantitative bite for (2) and (3)

- Attenuation effect increases with horizon
  - ϕ/ϕ^* decreases in T
  - $\phi/\phi^* \to 0$ as $T \to \infty$, even if $\lambda \approx 1$
  - for λ_c small enough, $\phi \to 0$ in absolute, not only relative to ϕ^*

A Numerical Illustration

- Modest friction: 25% prob that *others* failed to hear announcement
- All other parameters as in Gali's textbook



Fiscal Stimuli: Back- vs Front-Loading

- Standard NK prediction:
  - fiscal stimuli work because they trigger inflation
  - better to back-load so as to "pile up" inflation effects

- Our twist:
  - such piling up = iterating HOB
  - not as potent when CK assumption is dropped
  - rationale for front-loading: "minimize coordination friction"

Companion Work

Angeletos and Chen, "Dampening GE"

- Flexible formalization of GE attenuation
- Bridge gap between macro effects and micro elasticities
- Compare removing CK to dropping RE

Dropping RE vs Removing CK

- Cognitive discounting as in Gabaix (2016)
  - by assumption, subjective beliefs move less than rational expectations
  - can capture GE attenuation, but free to assume opposite
- Level-k Thinking as in Farhi and Werning (2017)
  - agents form beliefs by iterating on best responses, but stop before reaching the fixed point (which gives RE solution)
  - attenuation when GE amplifies PE, but *not* when GE offsets PE
- Our approach does not face these difficulties, plus:
  - immunity to Lucas critique
  - no conundrum with what agents do when they see that the actual outcomes are inconsistent with their beliefs
  - implies not only discounting but also backward-lookingness

Angeletos and Huo, "Anchored Expectations"

- Incomplete info = discounting + backward looking
- Application: NKPC
  - standard (without price indexation)

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t[\pi_{t+1}]$$

  - with incomplete info, it is as if

$$\pi_t = \kappa' x_t + \beta' \mathbb{E}_t[\pi_{t+1}] + \gamma \pi_{t-1}$$

$$\kappa' < \kappa, \quad \beta' < \beta, \quad \gamma > 0$$

  - i.e., micro-foundation of hybrid NKPC
- Other applications: micro-foundation of C habit and IAC

Conclusion

- Standard modeling has "overstated"
  - responsiveness of forward-looking expectations
  - potency of GE effects

- Applications:
  - lessen FG puzzle
  - rationale for front-loading fiscal stimuli
  - sluggish AD response to MP
  - anchored inflation expectations
  - Ricardian Equivalence
  - ....