Monetary Policy and the Redistribution Channel

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Abstract

This paper evaluates the role of redistribution in the transmission mechanism of monetary policy to consumption. Three channels affect aggregate spending when winners and losers have different marginal propensities to consume: an *earnings heterogeneity channel* from unequal income gains, a *Fisher channel* from unexpected inflation, and an *interest rate exposure channel* from real interest rate changes. Sufficient statistics from Italian and U.S. data suggest that all three channels are likely to amplify the effects of monetary policy. A standard incomplete markets model can deliver the empirical magnitudes if assets have plausibly high durations but a counterfactual degree of inflation indexation.

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1 Introduction

There is a conventional view that redistribution is a side effect of monetary policy changes, separate from the issue of aggregate stabilization which these changes aim to achieve. Most models of the monetary policy transmission mechanism implicitly adopt this view by featuring a representative agent. By contrast, in this paper I argue that redistribution is a *channel* through which monetary policy affects macroeconomic aggregates, because those who gain from accommodative monetary policy have higher marginal propensities to consume (MPCs) than those who lose. The simple argument goes back to Tobin (1982):

> Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and for debtors. But [...] the population is not distributed between debtors and creditors randomly. Debtors have borrowed for good reasons, most of which indicate a high marginal propensity to spend from wealth or from current income.

In this paper, I use consumer theory to refine Tobin's intuitions about aggregation. My analysis clarifies who gains and who loses from monetary policy changes, as well as the effect on aggregate consumption. Monetary expansions tend to increase real incomes, to raise inflation and to lower real interest rates. Not everyone is equally affected by these changes. This generates three distinct sources of redistribution.

First, monetary expansions induce gains in aggregate earnings from labor and profits. The distribution of these gains is unlikely to be equal: some agents benefit disproportionately, and conversely, some lose in relative terms. This is the *earnings heterogeneity channel* of monetary policy.

Second, unexpected inflation revalues nominal balance sheets, with nominal creditors losing and nominal debtors gaining: this is the *Fisher channel*, which has a long history in the literature since Fisher (1933). This channel has been explored by Doepke and Schneider (2006), who measure the balance sheet exposures of various sectors and groups of households in the United States to different inflation scenarios. Net nominal positions (NNPs) quantify the exposures to unexpected increases in the price level.

Real interest rate falls create a third, more subtle form of redistribution. These falls increase financial asset prices. But it is incorrect to claim that asset holders generally benefit: instead, we have to consider whether their assets have longer durations than their liabilities. Importantly, liabilities include consumption plans, and assets include human capital. Unhedged interest rate exposures (UREs) — the difference between all maturing assets and liabilities at a point in time — are the correct measure of households' balance-sheet exposures to real interest rate changes, just like net nominal positions are for price level changes. For example, agents whose financial wealth is primarily invested in short-term certificates of deposit tend to have positive UREs, while those with large long-term bond investments or adjustable-rate mortgage liabilities tend to have negative UREs. Real interest rate falls redistribute away from the first group towards the second group: this is what I call the *interest rate exposure channel*.

In this paper, I show how these three redistribution channels affect the transmission mechanism of monetary policy to consumption. My main theoretical result decomposes the consumption effect of a transitory change in monetary policy into a contribution from each of these channels, together with an *aggregate income* and a *substitution channel*. Representative-agent models only feature the latter two. My theorem shows that redistribution amplifies these effects, provided that winners from monetary expansions have higher MPCs than losers. The rest of the paper argues that this is likely to be the case in practice. Hence, the redistributive effects of monetary policy are important to understand its aggregate effect.[1]

In the first part of the paper, I establish my main decomposition by studying a general aggregation problem. In partial equilibrium, I consider an optimizing agent with a given initial balance sheet, who values nondurable consumption and leisure, and is subject to a transitory change in income, inflation and the real interest rate. I decompose his consumption response into a substitution effect and a wealth effect, and show that the latter is the product of his MPC out of income and a balance-sheet revaluation term in which NNPs and UREs appear. This result is robust to the presence of durable goods, incomplete markets, idiosyncratic risk, and (certain kinds of) borrowing constraints. In other words, the MPC out of a windfall income transfer

[1] My theorem applies to a broad class of general equilibrium models with heterogeneous agents, so it can be used to understand consumption in other contexts than that of monetary policy. At the same time, I am leaving a number of redistributive channels out of my analysis. First, I abstract away from aggregate risk, so cannot handle changes in risk premia, as in Brunnermeier and Sannikov (2010). Second, I do not model limited participation, so monetary policy cannot differentially affect participants and nonparticipants, as in the studies of Grossman and Weiss (1983), Rotemberg (1984) and others. Finally, since I assume that all assets are remunerated at the risk-free rate, my analysis does not address the unequal incidence of inflation due to larger cash holdings by the poor (Erosa and Ventura 2002; Albanesi 2007). These are all interesting dimensions along which the theory could be extended.

is a key determinant of the response of optimizing consumers to inflation– or real interest rate–induced changes in their balance sheets. This result generalizes previous findings by Kimball (1990) on the importance of MPCs in incomplete-markets consumption models.

I then sum across the individual-level predictions and exploit the fact that financial assets and liabilities net out in general equilibrium to obtain the first-order response of aggregate consumption to simultaneous transitory shocks to output, inflation, and the real interest rate. This response is the sum of five terms, reflecting the contributions from the two aggregate and the three redistributive channels mentioned above. Moreover, the magnitudes of the redistributive channels are given by sufficient statistics: the cross-sectional covariances between MPCs and exposures to each aggregate shock. Since the pioneering work of Harberger (1964), sufficient statistics have been used in public finance to evaluate the welfare effect of hypothetical policy changes in a way that is robust to the specifics of the underlying structural model (see Chetty 2009 for a survey). Mine are useful to evaluate the impact of hypothetical changes in macroeconomic aggregates on *aggregate consumption* in a similarly robust way. All that is required is information on household balance sheets, income and consumption levels, and their MPCs.

By further assuming that the elasticity of intertemporal substitution σ and the elasticity of relative income to aggregate income γ are constant in the population, I obtain a set of five estimable moments that summarize all we need to know about agents' heterogeneity to recover the aggregate elasticities of consumption to the real interest rate, the price level, and aggregate income. Contrary to σ (and perhaps γ), these sufficient statistics are not structural parameters: they are likely to vary over time and across countries. I set out to measure them in three separate surveys, covering different time periods, countries, and methods from the literature. I use a 2010 Italian survey containing a self-reported measure of MPC (Jappelli and Pistaferri 2014); the 1999-2013 waves of the U.S. Panel Survey of Income Dynamics, together with semi structural approach to identify the MPC out of transitory income shocks (Blundell et al. 2008); and the 2001–2002 waves of the U.S. Consumer Expenditure Survey, together with a method that exploits the randomized timing of tax rebates as a source of identification for MPC (Johnson et al. 2006).

Consider first the elasticity of consumption to the real interest rate. In a representative-agent world, this elasticity is due to intertemporal substitution. It is negative, and its magnitude depends on σ. I define a method for measuring UREs, and show that, in each of my three datasets, their covariance with MPCs is also negative. Through the lens of my theorem, this implies that the interest rate exposure channel acts in the same direction as the substitution channel, and with comparable magnitude provided that σ is around 0.1-0.2. Hence representative-agent analyses that abstract from redistribution fail to capture an important reason why real interest rates affect consumption.

Similarly, across datasets, the covariance between MPCs and NNPs is negative. This implies that consumption tends to rise with inflation as a result of the Fisher channel. However, when cast in terms of elasticities, the magnitude is small: an unexpected permanent 1% increase in the price level raises consumption today by no more than 0.1%. This suggests that nominal redistribution could be important in explaining why aggregate consumption increases in monetary expansions, though its contribution is likely to be modest.

Finally, in line with previous literature, I estimate the covariance between MPCs and incomes to be negative in the data. If, in addition, low-income agents disproportionately benefit from increases in aggregate income — as suggested by Coibion et al. 2017 — the earnings heterogeneity channel also amplifies the effects of monetary policy. Future work can build on my results by providing more precise estimates of these sufficient statistics, as well as keeping track of their evolution over time.

A nascent literature analyzes the effects of monetary policy in dynamic stochastic general equilibrium models with rich heterogeneity, matching various aspects of the cross-section such as the wealth distribution. Prominent examples include Gornemann et al. (2012), McKay et al. (2016), and Kaplan et al. (2016). These structural models overcome a number of important limitations of my sufficient statistics approach. They can study the role of investment, analyze the precise interaction between monetary and fiscal policy, and explore the effect of shocks that are persistent and/or announced in advance. Yet, as highlighted by Kaplan et al. (2016), a version of my main decomposition survives in these more complex models, shedding light on the underlying mechanisms. Moreover, sufficient statistics can discipline the construction of these models. By making sure that the model's sufficient statistics match the data, researchers can ensure that, even if the model is misspecified, its predictions for the response of consumption to shocks are consistent with the empirical evidence.

I illustrate this procedure by considering the sufficient statistics generated by a standard partial-equilibrium incomplete markets model, similar to the one used as a building block by the literature. Mine is a Bewley-Huggett-Aiyagari model with nominal, long-term, circulating private IOUs (as in Huggett 1993). Such a model features rich heterogeneity in MPCs, UREs, NNPs and incomes. I calibrate it to the U.S. economy and quantitatively evaluate, in its steady state, the size of my sufficient statistics. This exercise delivers three main insights.

First, in the model, the interest rate exposure channel has the same sign and comparable magnitude as it does in the data. However, this result relies crucially on long asset durations. If instead all assets are short term (a typical assumption in the literature), changes in real interest rates have very large redistributive effects. The intuition is as follows: under a shorter maturity structure, debtors — the high-MPC agents in the economy — roll over a larger fraction of their liabilities each period, and their consumption plans are therefore very sensitive to changes in those rates. The role of asset durations I uncover holds under any degree of shock persistence. It is consistent with the results of Calza et al. (2013), who find that consumption reacts much more strongly to identified monetary policy shocks in countries where mortgages predominantly have adjustable rates.[2]

Second, I find that the benchmark calibration of the model in which all assets are nominal displays a Fisher channel with the same sign as in the data, but with a much larger magnitude. This is because inflation redistributes along the asset dimension, which in this class of models is highly correlated with MPC. As a result, Bewley models with nominal assets tend to overstate the correlation between MPCs and NNPs that exists in the data. A model with real assets, or in which assets have a high degree of inflation indexation, is more consistent with the empirical evidence.[3]

Finally, in the model with short-term debt, changes in real interest rates have asymmetric effects. The sufficient statistic approach correctly predicts the effect of any increase in the real rate, but it overpredicts in the other direction. This asymmetry comes from the differential response of borrowers at their credit limit to rises and falls in income: while these borrowers save an important fraction of the gains they get from low interest rates, they are forced to cut spending steeply when interest rates rise. This could help explain the empirical finding that interest rate hikes tend

[2] See also Rubio (2011) and Garriga et al. (2016).

[3] My model also replicates the empirical covariance between MPCs and incomes but, since I assume that incomes are exogenous, I stop short of a full assessment of the earnings heterogeneity channel. A previous version of this paper examined this channel in general equilibrium.

to lower output by more than falls increase it (Cover 1992; de Long and Summers 1988; Tenreyro and Thwaites 2016).

This paper is motivated by an an extensive empirical literature documenting that MPCs are large and heterogenous in the population (see Jappelli and Pistaferri 2010 for a survey), and that they depend on household balance sheet positions.[4] Recently, di Maggio et al. (2017) have measured the consumption response of households to changes in the interest rates they pay on their mortgages. My theory shows that this paper quantifies an important leg of the redistribution channel of monetary policy.

Several papers have focused on the redistributive channels of monetary policy I highlight in isolation. Coibion et al. (2017) propose an empirical evaluation of the earnings heterogeneity channel by measuring how identified monetary policy shocks affect income inequality in the Consumer Expenditure Survey. The Fisher channel has received a great deal of attention in the literature following the work of Doepke and Schneider (2006). For example, on the normative side, Sheedy (2014) asks when the central bank should exploit its influence on the price level to ameliorate market incompleteness over the business cycle. On the positive side, Sterk and Tenreyro (2015) show that the Fisher channel can be a source of effects of monetary policy under flexible prices in a non-Ricardian model. The interest rate exposure channel has, by contrast, not received much attention in the context of monetary policy.[5]

The importance of MPC differences in the determination of aggregate demand is well understood by the theoretical literature on fiscal transfers.[6] MPC differences between borrowers and savers, in particular, have been explored as a source of aggregate effects from shocks to asset prices or to borrowing constraints.[7] In Farhi and Werning (2016b), MPCs enter as sufficient statistics for optimal macro-prudential interventions under nominal rigidities. None of these studies, however, focus on the role of MPC differences in generating aggregate effects of monetary policy.

[4] See for example Mian et al. 2013; Mian and Sufi 2014; Baker 2017 and Jappelli and Pistaferri 2014.

[5] Redistribution through real interest rates does play a prominent role, for example, in Bassetto (2014)'s study of optimal fiscal policy or in Costinot et al. (2014)'s study of dynamic terms of trade manipulation.

[6] See Galí et al. 2007; Oh and Reis 2012; Farhi and Werning 2016a; McKay and Reis 2016.

[7] See King 1994; Eggertsson and Krugman 2012; Guerrieri and Lorenzoni 2015; Korinek and Simsek 2016.

The remainder of the paper is structured as follows. Section 2 presents a partial equilibrium decomposition of consumption responses to shocks into substitution and wealth effects. Section 3 provides my aggregation result and discusses the monetary policy transmission mechanism with and without heterogeneity. Section 4 assesses the quantitative magnitudes of each of my redistribution channels by measuring sufficient statistics in three surveys. Finally, section 5 compares the sufficient statistics from the data to those of a Huggett model, shedding light on their structural determinants. Section 6 concludes.

2 Household balance sheets and wealth effects

In this section, I show how households' balance sheets shape their consumption and labor supply adjustments to a transitory macroeconomic shock. I first highlight the forces at play in a life-cycle labor supply model (Modigliani and Brumberg 1954; Heckman 1974) featuring perfect foresight and balance sheets with an arbitrary maturity structure. Balance sheet revaluations and marginal propensities to consume and work play a crucial role in determining both the welfare and the wealth effects of the shock (theorem 1). Under certain conditions, the positive results from theorem 1 survive the addition of idiosyncratic income uncertainty (theorem 2) and therefore apply to a large class of microfounded models of consumption behavior.

2.1 Perfect-foresight model

Consider a household with separable preferences over nondurable consumption $\{c_t\}$ and hours of work $\{n_t\}$.[8] I assume no uncertainty for simplicity: the same insights obtain when markets are complete, except with respect to the unanticipated initial shock. The household is endowed with a stream of real unearned income $\{y_t\}$. He has perfect foresight over the general level of prices $\{P_t\}$ and the path of his nominal wages $\{W_t\}$, and holds long-term nominal and real contracts. Time is discrete, but the horizon may be finite or infinite, so I do not specify it in the

[8] I present results for separable preferences because expressions for substitution elasticities take simple and familiar forms in this case, but many of my results extend to arbitrary non satiable preferences (see Appendix A.3). I assume that both u and v are increasing and twice continuously differentiable, with u concave and v convex.

summations. The agent solves the following utility maximization problem:

$$\begin{aligned}\max \quad & \sum_t \beta^t \left\{u\left(c_t\right) - v\left(n_t\right)\right\} \\ \text{s.t.} \quad & P_t c_t = P_t y_t + W_t n_t + \left({}_{t-1}B_t\right) + \sum_{s\geq 1} \left({}_tQ_{t+s}\right)\left({}_{t-1}B_{t+s} - {}_tB_{t+s}\right) \\ & \qquad + P_t\left({}_{t-1}b_t\right) + \sum_{s\geq 1}\left({}_tq_{t+s}\right) P_{t+s}\left({}_{t-1}b_{t+s} - {}_tb_{t+s}\right) \qquad (1)\end{aligned}$$

The flow budget constraint (1) views the consumer, in every period t, as having a portfolio of zero coupon bonds inherited from period $t-1$, and determining consumption c_t, labor supply n_t, as well as the portfolio of bonds he chooses to carry into the next period.[9] Specifically, ${}_tQ_{t+s}$ is the time-t price of a nominal zero-coupon bond paying at $t+s$, ${}_tq_{t+s}$ the price of a real zero-coupon bond, and ${}_tB_{t+s}$ (respectively ${}_tb_{t+s}$) denote the quantities purchased. This asset structure is the most general one that can be written for this dynamic environment with no uncertainty. To keep the problem well-defined, I assume that the prices of nominal and real bonds prevent arbitrage profits. This implies a Fisher equation for the nominal term structure:

$${}_tQ_{t+s} = \left({}_tq_{t+s}\right)\frac{P_t}{P_{t+s}} \quad \forall t, s$$

I focus on the period $t = 0$. The environment allows for a very rich description of the household's initial holdings of financial assets, denoted by the consolidated claims, nominal $\left\{{}_{-1}B_t\right\}_{t\geq 0}$ and real $\left\{{}_{-1}b_t\right\}_{t\geq 0}$, due in each period. The former could represent deposits, long-term bonds and most typical mortgages. The latter could represent stocks (which here pay a riskless real dividend stream and therefore are priced according to the risk-free discounted value of this stream), inflation-indexed government bonds, and price-level adjusted mortgages. I write the real wage at t as $w_t \equiv \frac{W_t}{P_t}$, the initial real term structure as $q_t \equiv \left({}_0q_t\right)$, and impose the present-value normalization $q_0 = 1$.

Using either a terminal condition if the economy has finite horizon, or a transversality condition if the economy has infinite horizon, the flow budget constraints consolidate into an intertemporal budget constraint:

$$\sum_{t\geq 0} q_t c_t = \underbrace{\sum_{t\geq 0} q_t\left(y_t + w_t n_t\right)}_{\omega^H} + \underbrace{\sum_{t\geq 0} q_t\left(\left({}_{-1}b_t\right) + \left(\frac{{}_{-1}B_t}{P_t}\right)\right)}_{\omega^F} \equiv \omega \qquad (2)$$

[9] He may, of course, just decide to roll over his position from the previous period. This corresponds to the costless trade that sets ${}_{t-1}b_{t+s} = {}_tb_{t+s}$ and ${}_tB_{t+s} = {}_{t-1}B_{t+s}$ for all s.

Equation (2) states that the present value of consumption must be equal to wealth ω: the sum of human wealth ω^H (the present value of all future income) and financial wealth ω^F. Since $\{_{-1}B_t\}$ and $\{_{-1}b_t\}$ only enter (2) through ω^F, it follows that financial assets with the same initial present value deliver the same solution to the consumer problem. For instance, this framework predicts that a household with an adjustable-rate mortgage (ARM), with $_{-1}B_0 = -L$, chooses the same plan for consumption and labor supply as an otherwise identical household with a fixed-rate mortgage (FRM), $_{-1}B_t = -M$ for $t = 0 \ldots T$, provided the two mortgages have the same outstanding principal, i.e. $L = \sum_{t=0}^{T} Q_t M$. In this sense, the composition of balance sheets is irrelevant. But this composition matters following a shock, as the next section shows.

2.2 Adjustment after a transitory shock

I now consider an exercise where, keeping balance sheets fixed at $\{_{-1}B_t\}_{t\geq 0}$ and $\{_{-1}b_t\}_{t\geq 0}$, the paths of variables relevant to the consumer choice problem change in the following way:

a) all nominal prices rise in proportion, $\frac{dP_t}{P_t} = \frac{dP}{P}$, for $t \geq 0$
b) all present-value real discount rates rise in proportion, $\frac{dq_t}{q_t} = -\frac{dR}{R}$, for $t \geq 1$
c) the Fisher equation holds at the new sequence of prices: $\frac{dQ_t}{Q_t} = -\frac{dR}{R}$ for $t \geq 1$
d) the agent's unearned income at $t = 0$ rises by dy, and his real wage by dw.

This particular variation, depicted in figure 1, captures in a stylized way the major changes in a consumer's environment that usually follow a temporary change in monetary policy: over a period labelled $t = 0$, incomes and wages increase, the price level rises due to inflation between $t = -1$ and $t = 0$, and the real interest rate $R_0 = \frac{q_0}{q_1}$ falls.[10] As I show formally in Appendix A.1, these are the changes that occur in the standard representative-agent New Keynesian model following a one-period change in monetary policy. Hence this variation is a natural starting point for an analysis of the effects of monetary policy on individual households.

[10] The assumption that balance sheets are fixed implies that coupon payments are not contingent on the macroeconomic changes dw, dy, dP or dR. This is an incomplete markets assumption. If assets payoffs are state contingent, my results go through provided the change in payoff is counted towards dy.

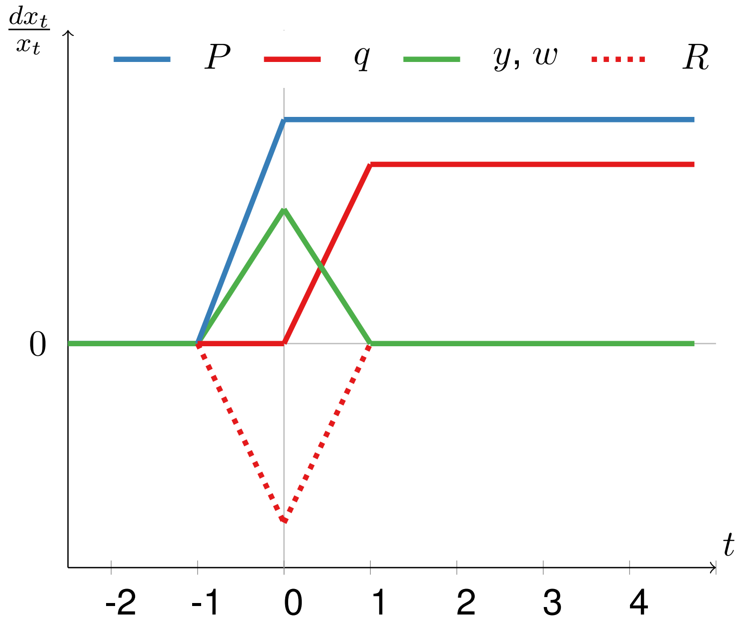

Figure 1: The experiment

I am interested in the first-order change in initial consumption $dc \equiv dc_0$, labor supply $dn \equiv dn_0$, and welfare dU that results from this change in the environment.

Let σ and ψ be the local Frisch elasticities of substitution in consumption and hours.[11] Define the marginal propensity to consume as $MPC = \frac{\partial c_0}{\partial y_0}$ along the initial path. When a consumer exogenously receives an extra dollar of income, he increases consumption by MPC dollars, but, to the extent that labor supply is elastic ($\psi > 0$), he also reduces hours by $MPN = \frac{\partial n_0}{\partial y_0} < 0$, leaving only $MPS = 1 - MPC + w_0 MPN$ dollars for saving.[12]

These behavioral responses to income changes turn out to also matter for the response to the real interest rate, wage, and price level changes, as the following theorem shows.

[11] Formally, $\sigma \equiv -\frac{u'(c_0)}{c_0 u''(c_0)} > 0$ and $\psi \equiv \frac{v'(n_0)}{n_0 v''(n_0)} \geq 0$.

[12] Indeed, separable utility guarantees that $MPC \in (0,1)$, $MPS \in (0,1)$ and $MPN \leq 0$: in other words, consumption, saving and leisure are 'normal'. Below I provide an alternative definition of the marginal propensity to consume that corresponds to the more familiar split between consumption and savings alone.

Theorem 1. *To first order, dropping $t = 0$ subscripts whenever unambiguous,*

$$dc = MPC\left(d\Omega + \psi n dw\right) - \sigma c MPS \frac{dR}{R} \tag{3}$$

$$dn = MPN\left(d\Omega + \psi n dw\right) + \psi n MPS \frac{dR}{R} + \psi n \frac{dw}{w} \tag{4}$$

$$dU = u'(c)\, d\Omega \tag{5}$$

where $d\Omega$, the net-of-consumption wealth change, is given by

$$d\Omega = dy + ndw + \underbrace{\left(y + wn + \left(\frac{{}_{-1}B_0}{P_0}\right) + \left({}_{-1}b_0\right) - c\right)}_{\text{Unhedged interest Rate Exposure } (URE)} \frac{dR}{R} - \underbrace{\sum_{t\geq 0} Q_t \left(\frac{{}_{-1}B_t}{P_0}\right)}_{\text{Net Nominal Position } (NNP)} \frac{dP}{P} \tag{6}$$

The theorem, proved in appendix A.2, follows from an application of Slutsky's equations — separating the wealth and the substitution effects that result from the shock. The relative price changes dR and dw generate substitution effects on consumption and labor supply with familiar signs, and magnitudes given by a combination of Frisch elasticities and marginal propensities. All wealth effects get aggregated into a net term, $d\Omega$, which affects consumption and labor supply after multiplication by the marginal propensity to consume and work, respectively.

Note that theorem 1 makes no assumption on horizon or the form of u and v. In appendix A.3, I show that it extends to general utility functions and to persistent shocks.

Net wealth revaluation: determinants and implications. The net wealth change $d\Omega$ in (6) is the key expression determining the sign and the magnitude of the welfare and the wealth effects in theorem 1. This term is a sum of products of *balance-sheet exposures* by *changes in aggregates*. The exposure to a one-off, immediate increase in the price level is the negative of the present value of the household's net nominal assets, also known as their net nominal position (NNP). This term can be computed directly from a survey of the household's finances. Doepke and Schneider (2006) conduct this exercise for various groups of U.S. households and show that NNPs are large and heterogenous in the population: they are very positive for rich, old households and negative for the young middle class with mortgage debt. Theorem 1 shows that these numbers are not only relevant for welfare, but also for the consumption response to this inflation scenario. Clearly, the composition of balance sheets matters. A household with a positive NNP loses when the price level increases. This exposure can be avoided by investing all wealth in inflation-indexed instruments, that is, by letting ${}_{-1}B_t = 0$ for all t.[13]

Just as an change in the price level 'acts' upon the consumer's net nominal position, equation (6) shows that a change in the real interest rate acts upon what I call his *unhedged interest rate exposure,* or URE. URE is the difference between *all* maturing assets (including income) and liabilities (including planned consumption) at time 0. It represents the net saving requirement of the household at time 0, from the point of view of date -1. Because it includes the stocks of financial assets that mature at date 0 rather than interest flows, it can significantly diverge from traditional measures of savings, in particular if investment plans have short durations.

Why does URE determine the wealth effect following a real interest rate change dR at time 0? To fix ideas, suppose $dR > 0$. This increase in the discount rate reduces the present value of assets, but also the present value of liabilities, with consumption being one such liability. The result is a net wealth loss if future assets exceed future liabilities. But this can only happen if currently-maturing liabilities exceed currently-maturing assets, i.e. if $URE < 0$. Indeed, equation (2) implies that

$$\sum_{t\geq 1} q_t \left(y_t + w_t n_t\right) + \sum_{t\geq 1} q_t \left(\left({}_{-1}b_t\right) + \left(\frac{{}_{-1}B_t}{P_t}\right)\right) - \sum_{t\geq 1} q_t c_t = -URE$$

The intuition here is that a fall in the price of future consumption relative to current consumption is the same as an increase in the price of current consumption relative to future consumption. But a rise in the price of current goods benefits those consumers that are supplying more goods than they demand at that date, and conversely, it hurts the net buyers of current goods. URE is the measure of the net exposure to this price change. Note that URE is *also* measurable from a survey of household finances that has information on income and consumption.[14]

This observation has the important implication that the *duration of asset plans* matters to determine what happens after a change in real interest rates. Fixed rate mortgage holders and annuitized retirees usually have income and outlays roughly balanced, and hence a URE of about zero. By contrast, ARM holders tend to

[13] If prices adjust more sluggishly, the Fisher exposure measure changes. For example, if prices adjust only after T (so that $\frac{dP_t}{P_t} = \frac{dP}{P}$ for $t \geq T$), the formulas hold if NNP is replaced by $\sum_{t\geq T} Q_t \left(\frac{{}_{-1}B_t}{P_0}\right)$, the present value of assets maturing after T. In this case, short-maturity nominal assets maintain constant value, while long-maturity assets decline in value due to the increase in nominal discount rates that follows the expected rise in inflation. The general expression for any given path of price adjustment is given by formula (A.37) in appendix A.3.

[14] By contrast, measuring the exposure to real interest rate changes at any future date requires the knowledge of future income and consumption plans.

have negative URE, and savers with large amounts of wealth invested at short durations tend to have positive URE. Hence the theory predicts that the former tend to lose and the latter tend to gain from a temporary increase in real interest rates. In response, consumption falls whenever the substitution effect dominates the wealth effect. Equation 3 allows us to quantify these two effects, and shows that this happens whenever $\sigma c MPS \geq MPC \cdot URE$.

Monetary policy and household welfare. Theorem 1 shows that asset value changes give incomplete information to understand the effects of monetary policy on household welfare. In the model just presented, monetary policy can be thought of as influencing asset values through three channels: a risk-free real discount rate effect (dR), an inflation effect (dP), and an effect on dividends (dy). But these asset value changes do not enter $d\Omega$ directly, so they are not relevant *on their own* to understand who gains and who loses from monetary policy, contrary to what popular discussions sometimes imply. For example, it is sometimes argued that accommodative monetary policy benefits bondholders by increasing bond prices. Yet theorem 1 shows that, while increases in dividends do raise welfare, lower real risk-free rates have ambiguous effects on savers. They have no effect on bondholders whose dividend streams initially match the difference between their target consumption and other sources of income. They benefit households who hold long-term bonds to finance short-term consumption, through the capital gains they generate. And they hurt households who finance a long consumption stream with short-term bonds, by lowering the rates at which they reinvest their wealth. Unhedged interest rate exposures, not asset price changes, constitute the welfare-relevant metric for the impact of real interest rate changes on households. This is why it is important to measure them, which I do in section 4.

The response of consumption to overall income changes. Theorem 1 draws a distinction between exogenous changes in income and changes in wages, since the latter have substitution effects on consumption. However, since preferences are separable, it is possible to rewrite the consumption response as a function of the *total* income change, inclusive of the labor supply response, as shown in appendix A.4.

Corollary 1. *Given an overall change in income $dY = dy + ndw + wdn$, the household's consumption response is given by*

$$dc = \hat{MPC}\left(dY + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \sigma c\left(1 - \hat{MPC}\right)\frac{dR}{R} \tag{7}$$

where $\hat{MPC} = \frac{MPC}{MPC+MPS} = \frac{MPC}{1+wMPN} \geq MPC.$

Hence, once we have factored in the endogenous response of income to transfers, the relevant marginal propensity to consume becomes $\hat{MPC}$, the number between 0 and 1 that determines how the remaining amount of income is split between consumption and savings. This corresponds more closely to the textbook measure of the marginal propensity to consume. It is also what empirical measures tend to pick up, since these are usually regressions of observed consumption on observed income.[15]

Durable goods. So far I have restricted my analysis to nondurable consumption. However, durable expenditures tend to account for a substantial share of the overall consumption response to monetary policy shocks, so it is important to understand their behavior. Understanding how durable goods fit into the theory is also important to deliver an accurate map to consumption data. As I show formally in Appendix A.5, adding durable goods to the model does not alter the substantive conclusions from Theorem 1, but there are some subtleties.

The most straightforward case is the one in which the relative price of durable goods and nondurable goods is constant. In this case, formulas (3) or (7) continue to hold, provided that c is interpreted as overall expenditures, MPC is the marginal propensity to spend on all goods, URE subtracts durable expenditures, and σ is adjusted upwards to reflect the fact that durable goods allow more opportunities for intertemporal substitution.

In multi-sector New Keynesian models with durable goods, a constant relative price of durable goods obtains when the prices of durables and nondurables are equally sticky (Barsky et al. 2007). However, there is some evidence that durables have more flexible prices (Klenow and Malin 2010), in which case the models imply a negative comovement between the relative price of durables p and the nondurable

[15] Note that, if hours affected the marginal utility of consumption, it would not be generally possible to obtain an expression such as (7). Instead, dw would enter separately, with a sign reflecting the degree of complementarity between consumption and labor supply.

real interest rate R. Let $\epsilon = -\frac{\partial p}{p}\frac{R}{\partial R}$ be the corresponding elasticity — in this case, nondurables and durables matter separately, so there no longer exists a straightforward notion of aggregate demand. Instead, in Appendix A.5 I derive separate expressions for the change in nondurable and durable consumption as a function of ϵ. These resemble equations (3) or (7), except for the fact that the expression for URE only subtracts a share $1-\epsilon$ of durable expenditures.[16] Consider a given size increase in the nondurable real interest rate dR. As ϵ rises, durable prices fall by more, and durable demand tends to expand. This is counterfactual, as argued by Barsky et al. (2007). Hence, in practice, elasticities closer to 0 may be more reasonable. In the empirical section, I will assume $\epsilon = 0$ as a benchmark from computing UREs, but I will also consider robustness of my results to the value of ϵ.

Even though all the results presented in this section assume no uncertainty and perfect foresight, they apply directly to environments with uncertainty provided that markets are complete, *except* for the shock that is unexpected (all summations are then over states as well as dates). An important feature of all these environments is that the marginal propensity to consume, MPC, is the same out of all forms of wealth ($\frac{\partial c_0}{\partial y_0} = \frac{\partial c_0}{\partial \omega}$). The next section relaxes this assumption.

2.3 THE CONSUMPTION RESPONSE TO SHOCKS UNDER INCOMPLETE MARKETS

I now consider a dynamic, incomplete-market partial equilibrium consumer choice model. The consumer faces an idiosyncratic process for real wages $\{w_t\}$ and unearned income $\{y_t\}$. He chooses consumption c_t and labor supply n_t to maximize the separable expected utility function

$$\mathbb{E}\left[\sum_t \beta^t \left\{u\left(c_t\right) - v\left(n_t\right)\right\}\right] \tag{8}$$

The horizon is still not specified in the summation. As in the previous section, it will only influence behavior through its impact on the *MPC*. To model market incompleteness in a general form, I assume that the consumer can trade in N stocks as well as in a nominal long-term bond. In period t, stocks pay real dividends $\mathbf{d}_t = (d_{1t} \ldots d_{Nt})$ and can be purchased at real prices $\mathbf{S}_t = (S_{1t} \ldots S_{Nt})$; the consumer's portfolio of shares is denoted by θ_t. Following the standard formulation in

[16] When $\epsilon = 1$, durable purchases are not counted at all in URE, for the same reason that purchases of bonds or shares aren't: in this case, durables completely hedge real interest rate movements.

the literature, I assume that the long-term bond can be bought at time t at price Q_t and is a promise to pay a geometrically declining nominal coupon with pattern $(1, \delta, \delta^2, \ldots)$ starting at date $t+1$. The current nominal coupon, which I denote Λ_t, then summarizes the entire bond portfolio, so it is not necessary to separately keep track of future coupons. The household's budget constraint at date t is now

$$P_t c_t + Q_t \left(\Lambda_{t+1} - \delta \Lambda_t\right) + \theta_{t+1} \cdot P_t \mathbf{S}_t = P_t y_t + P_t w_t n_t + \Lambda_t + \theta_t \cdot \left(P_t \mathbf{S}_t + P_t \mathbf{d}_t\right) \tag{9}$$

A borrowing constraint limits trading. This constraint specifies that real end-of-period wealth cannot be too negative: specifically,

$$\frac{Q_t \Lambda_{t+1} + \theta_{t+1} \cdot P_t \mathbf{S}_t}{P_t} \geq -\frac{\overline{D}}{R_t} \tag{10}$$

for some $\overline{D} \geq 0$, where R_t is the real interest rate at time t. The constraint in (10) is a standard specification for borrowing limits[17] and we will see that it generates reactions of constrained agents to balance sheet revaluations that are closely related to those of unconstrained agents. Given that the extent to which borrowing constraints react to macroeconomic changes is an open question, (10) provides an important benchmark.

Provided that the portfolio choice problem just described has a unique solution at date $t-1$, the household's net nominal position and his unhedged interest rate exposure are both uniquely pinned down in each state at time t. This contrasts with the environment in section 2.2, where the consumer was indifferent between all portfolio choices. Here, these quantities are defined as

$$\begin{aligned} NNP_t &\equiv \left(1 + Q_t \delta\right) \frac{\Lambda_t}{P_t} \\ URE_t &\equiv y_t + w_t n_t + \frac{\Lambda_t}{P_t} + \theta_t \cdot \mathbf{d}_t - c_t \end{aligned}$$

As before, NNP_t is the real market value of nominal wealth: the sum of the current coupon, Λ_t, and the value of the bond portfolio if it were sold immediately, $Q_t \delta \Lambda_t$. Similarly, URE_t is maturing assets (including income, real coupon payments and dividends) net of maturing liabilities (including consumption).

Consider the predicted effects on consumption resulting from a simultaneous unexpected change in his current unearned income dy, his current real wage dw, the general price level dP and the real interest rate dR, for one period only. Assume that this variation leads asset prices to adjust to reflect the change in discounting

[17] For example, with short-term debt and no stocks ($N = \delta = 0$), $Q_t = \frac{1}{R_t} \frac{P_t}{P_{t+1}}$ and (10) reads $\frac{\Lambda_{t+1}}{P_{t+1}} \geq -\overline{D}$, as in Eggertsson and Krugman (2012).

alone : $\frac{dQ}{Q} = \frac{dS_j}{S_j} = -\frac{dR}{R}$ for $j = 1 \ldots N$.[18] If $MPC = \frac{\partial c}{\partial y}$, and both MPN and MPS are similarly defined as the responses to current *income* transfers, then the positive results from theorem 1 carry through.

Theorem 2. *Assume that the consumer is at an interior optimum, at a binding borrowing constraint, or unable to access financial markets (in the latter two cases, let MPS=0). Then his first order change in consumption dc and labor supply dn continue to be given by equations (3) and (4). In particular, writing $\hat{MPC} \equiv \frac{MPC}{MPC+MPS}$, the relationship between dc and the total change in income $dY = dy + ndw + wdn$ is still given by equation (7).*

The proof is given in appendix A.6. The intuition for why MPC, MPN and MPS are relevant to understand the response of all agents to changes in the real interest rate and the price level is simple: when the consumer is locally optimizing, these quantities summarizes the way in which he reacts to all balance-sheet revaluations, income being only one such revaluation. When the borrowing limit *is* binding, consumption and labor supply adjustments depend on the way the borrowing limit changes when the shock hits. Under the specification (10), the changes in dR and dP free up borrowing capacity[19] exactly in the amount $URE\frac{dR}{R} - NNP\frac{dP}{P}$. Finally, when the consumer is unable to access financial markets, he lives hand-to-mouth so $NNP = URE = 0$. In these latter two cases, $\hat{MPC} = 1$ so we can interpret the consumption response as a pure wealth effect.

By showing that the marginal propensity to consume out of transitory income shocks, which has been the focus of a large empirical literature, remains a key sufficient statistic for predicting behavior with respect to other changes in consumer balance sheets, theorem 2 provides important theoretical restrictions. The rest of the paper takes these restrictions as given and uses them to predict aggregate consumption responses to changes in R or P. But these restrictions are also directly testable empirically: given independent variation in dP, dy and dR as well as individual balance sheet information, one could check that individual consump-

[18] This is a natural assumption that obtains if asset prices are determined in a general equilibrium with incomplete markets. Absence of arbitrage in such a model implies the existence of a probability measure $\mathbb{Q}$ such that the price of each stock j at date 0 is $S_{0j} = \frac{1}{R_0}\mathbb{E}^{\mathbb{Q}}\left[\sum_{t\geq 1} \frac{1}{R_1 \ldots R_{t-1}} d_{jt}\right]$, where R_t is the sequence of risk-free rates. My variation affects R_0 but does not affect future interest rates, dividends, or risk-neutral probabilities, so results in $\frac{dS_{0j}}{S_{0j}} = -\frac{dR}{R}$. The argument for $\frac{dQ_0}{Q_0} = -\frac{dR}{R}$ is identical.

[19] The form of the borrowing constraint is clearly important for this result. For example, if the constraint on the level of wealth (10) is replaced by a constraint on the flow of income received from financial markets, $\frac{Q_t \Lambda_{t+1} + \theta_{t+1} \cdot P_t \mathbf{S}_t}{P_t} - \frac{\delta Q_t \Lambda_t + \theta_t \cdot P_t \mathbf{S}_t}{P_t} \geq -\overline{D}$, then the result collapses to $dc = dY$.

tion responds in accordance with equations (3) or (7). This provides an interesting avenue for future empirical work on consumption behavior.

3 Aggregation and the redistribution channel

This section shows how the microeconomic demand responses derived in section 2 aggregate in general equilibrium to explain the economy-wide response to shocks in a large class of heterogenous-agent models (theorem 3).

3.1 Environment

Consider a closed economy populated by I heterogenous types of agents with separable preferences (8). Each agent type i has its own discount factor β_i, period utility functions u_i and v_i, and time horizon. To accommodate idiosyncratic uncertainty, assume that within each type i there is a mass 1 of individuals, each in an idiosyncratic state $s_{it} \in S_i$. I write $\mathbb{E}_I [z_{it}]$ for the cross-sectional average of any variable z_{it}, taken over individual types I and idiosyncratic states S_i. I write all aggregate variables in per capita units, so for example aggregate (per capita) consumption C_t is equal to average individual consumption $\mathbb{E}_I [c_{it}]$.

Agents and asset structure. Each agent type i in state s_{it} has a stochastic endowment of $e_i (s_{it})$ efficient units of work, and receives a wage of $w_{it} = e_i (s_{it}) w_t$ per hour, where w_t is the real wage per efficient hour. By choosing n_{it} hours of work, he therefore receives $w_t e_{it} n_{it}$ in earned income. The agent also receives unearned income $y_{it} = d_{it} - t_{it}$, the total dividends on the trees he owns d_{it} net of taxes from the government t_{it}. Let the agent's overall gross-of-tax income be

$$Y_{it} \equiv w_t e_{it} n_{it} + d_{it}. \tag{11}$$

The economy has a fixed supply of aggregate capital K. A set of N trees constitute claims to firm profits and the capital stock. Each tree delivers dividends which, in the aggregate, add up to the sum of aggregate capital income and profits: $\mathbb{E}_I [d_{it}] = \rho_t K + \pi_t$. Agents can also trade nominal government bonds in net supply B_t, as well as a set of $J - 1$ additional assets in zero net supply that can be nominal or real. Each agent of type i can trade a subset N_i of the trees and a subset J_i of the other assets. If both N_i and J_i are empty, agents of type i live hand-to-mouth. In other cases, I assume that trading is subject to a type-specific borrowing constraint $\overline{D_i}$, which takes the form in (10) and may be infinite.

Firms. There exists a competitive firm producing the unique final good in this economy, in quantity Y_t and nominal price P_t, by aggregating intermediate goods with a constant-returns technology. These intermediate goods are produced by a unit mass of firms j under constant returns to scale, using the production functions $X_{jt} = A_{jt}F(K_{jt}, L_{jt})$. Markets for inputs are perfectly competitive, so firms take the real wage w_t and the real rental rate of capital ρ_t as given. These firms sell their products under monopolistic competition and their prices can be sticky. Firm j therefore sets its price P_{jt} at a markup over marginal cost and make real profits π_{jt}.[20] Summing across firms $j \in J$, aggregate production is equal to aggregate income:

$$Y_t = \mathbb{E}_J\left[\frac{P_{jt}}{P_t}X_{jt}\right] = w_t\mathbb{E}_J[L_{jt}] + \rho_t\mathbb{E}_J[K_{jt}] + \mathbb{E}_J[\pi_{jt}] \tag{12}$$

Government. A government has nominal short-term debt B_t, spends G_t, and runs the tax-and-transfer system. Its nominal budget constraint is therefore:

$$Q_tB_{t+1} = P_tG_t + B_t - P_t\mathbb{E}_I[t_{it}] \tag{13}$$

where $Q_t = \frac{1}{R_t}\frac{P_t}{P_{t+1}}$ is the one-period nominal discount rate. The consequences of price-induced redistributive effects between households and the government depend crucially on the fiscal rule. I assume a simple rule in which the government targets a constant real level of debt $\frac{B_t}{P_t} = \overline{b} > 0$ and spending $G_t = \overline{G} > 0$. I also assume that the government balances its budget at the margin by adjusting all transfers in a lump-sum manner. Hence, unexpected increases in P_t (which create ex-post deviations of $\frac{B_t}{P_t}$ from $\overline{b}$) and reductions in the real interest rate R_t result in immediate lump-sum rebates.

[20] Specifically, if μ_{jt} is firm j's markup at time t, then $\pi_{jt} = (\mu_{jt} - 1)(w_tL_{jt} + \rho_tK_{jt})$.

Market clearing. In equilibrium, the markets for capital, labor and goods all clear. This implies that at all times t

$$\mathbb{E}_J[K_{jt}] \equiv K \tag{14}$$

$$\mathbb{E}_I[e_{it}n_{it}] = \mathbb{E}_J[L_{jt}] \tag{15}$$

$$\mathbb{E}_I[Y_{it}] = Y_t = C_t + G_t \tag{16}$$

Equilibrium also implies market clearing in all $J+N$ asset markets. This environment nests a large class of one-good, closed economy general equilibrium models. It can accommodate many assumptions about population structure, asset market structure and participation, heterogeneity in preferences, endowments and skills, as well as the nature of price stickiness. With some minor modifications, it would accommodate wage stickiness as well.

3.2 Aggregation result

I am interested in the aggregate consumption response to a perturbation of this environment in which individual gross incomes dY_i, nominal prices dP and the real interest rate dR change at $t=0$ only. This exercise is useful to understand the effect of an *unexpected shock* that has *no persistence*. Let $dY \equiv \mathbb{E}_I[dY_i]$ be the aggregate change in gross income. Assuming labor market clearing after the shock, this is also the aggregate output change.

Aggregation is simplified by several restrictions from market clearing at $t=0$. Market clearing for nominal assets implies that all nominal positions net out except for that of the government,

$$\mathbb{E}_I[NNP_{it}] = \overline{b} = -NNP_{gt} \quad \forall t \tag{17}$$

and market clearing for all assets, combined with (11) — (16) implies[21] that

$$\mathbb{E}_I[URE_{it}] = Y_t - \mathbb{E}_I[t_{it}] + \frac{B_t}{P_t} - C_t = G_t + \frac{B_t}{P_t} - \mathbb{E}_I[t_{it}] = -URE_{gt} \tag{18}$$

where NNP_{gt} and URE_{gt} are naturally defined as the net nominal position and the unhedged interest rate exposure of the government sector. Equations (17) and (18) are crucial restrictions from general equilibrium: since one agent's asset is another's liability, net nominal positions and interest rate exposures must net out in a closed

[21] To see this, note that if b_{it} denotes the asset coupons that mature at time t for household i, we have $URE_{it} = Y_{it} - t_{it} + b_{it} - c_{it}$. Using market clearing in the $J-1$ zero net supply assets, all these coupons net out except for the government coupon, which here is $\mathbb{E}_I[b_{it}] = \frac{B_t}{P_t}$. The result then follows from goods market clearing and the government budget constraint.

economy. Aggregation of consumer responses as described by theorem 2 shows that the per capita aggregate consumption change can be decomposed as the sum of five channels:

Theorem 3. *To first order, in response to dY_i, dY, dP and dR, aggregate consumption changes by*

$$dC = \underbrace{\mathbb{E}_I\left[\frac{Y_i}{Y}M\hat{P}C_i\right]dY}_{\text{Aggregate income channel}} + \underbrace{\text{Cov}_I\left(M\hat{P}C_i, dY_i - Y_i\frac{dY}{Y}\right)}_{\text{Earnings heterogeneity channel}} - \underbrace{\text{Cov}_I\left(M\hat{P}C_i, NNP_i\right)\frac{dP}{P}}_{\text{Fisher channel}}$$
$$+\left(\underbrace{\text{Cov}_I\left(M\hat{P}C_i, URE_i\right)}_{\text{Interest rate exposure channel}} - \underbrace{\mathbb{E}_I\left[\sigma_i\left(1-M\hat{P}C_i\right)c_i\right]}_{\text{Substitution channel}}\right)\frac{dR}{R} \quad (19)$$

The proof is given in appendix A.7. The key step is to aggregate predictions from theorem 2, decomposing i's individual income change as $dY_i = \frac{Y_i}{Y}dY + dY_i - \frac{Y_i}{Y}dY$ (the sum of an aggregate component and a redistributive component), and using market clearing conditions, the fiscal rule, and the fact that $\mathbb{E}_I\left[dY_i - \frac{Y_i}{Y}dY\right] = 0$ to transform expectations of products into covariances.

Theorem 3 shows that, in the class of environments I consider, a small set of sufficient statistics is enough to understand and predict the first-order response of aggregate consumption to a macroeconomic shock. Equation (19) holds irrespective of the underlying model generating MPCs and exposures at the micro level, as well as the relationship between dY, dP and dR at the macro level. Most of the bracketed terms are cross-sectional moments that are measurable in household level micro-data and are informative about the economy's macroeconomic response to a shock, no matter the source of this shock. The two exceptions are the EISs σ_i, which need to be obtained from other sources, and $dY_i - Y_i\frac{dY}{Y}$, which in general depends on the driving force behind the change in output.

I now use this theorem to discuss the channels of monetary policy transmission under heterogeneity. Alternative applications, for example to short-term redistributive fiscal policy or open-economy models, are also possible.

3.3 Monetary policy shocks with and without a representative agent

Consider a transitory, accommodative monetary policy shock that, as in figure 1, lowers the real interest rate and raises aggregate income for one period ($dR < 0$, $dY > 0$), and permanently raises the price level ($\frac{dP}{P} > 0$). Since these are the changes implied by the textbook New Keynesian model with sticky prices and flexible wages after a transitory monetary policy shock, we can apply theorem 3 to understand the consumption response in that model.

The textbook model features a representative agent ($I = 1$) with separable preferences and EIS σ. Hence all covariance terms in (19) are zero, and we are left with

$$dC = M\hat{P}CdY - \sigma\left(1 - M\hat{P}C\right)C\frac{dR}{R} \tag{20}$$

The first term in (20) is a general-equilibrium income effect, and the second term is a substitution effect.[22] Solving out for $dC = dY$ gives the textbook response, $\frac{dC}{C} = -\sigma\frac{dR}{R}$. Intuitively, a Keynesian multiplier $\frac{1}{1-M\hat{P}C}$ amplifies the initial 'first-round' effect from intertemporal substitution. Here this multiplier is entirely microfounded, and in particular takes into account the substitution and wealth effects on labor supply that play out in the background.

Heterogeneity implies a role for redistributive channels in the monetary transmission mechanism, except under special conditions. For example, if aggregate income is distributed proportionally to individual income, so that $dY_i = \frac{Y_i}{Y}dY$; if no equilibrium asset trade is possible, so that agents consume all their incomes $Y_i = c_i$ and $NNP_i = URE_i = 0$; and if all agents have the same elasticity of intertemporal substitution $\sigma_i = \sigma$, then the representative-agent response $\frac{dC}{C} = -\sigma\frac{dR}{R}$ obtains even under heterogeneity. This important neutrality result is studied in Werning (2015).

Away from this benchmark, the redistributive channels of monetary policy can be signed and quantified by measuring the covariance terms in equation (19), either directly in micro data or within a given model. I follow each of these routes in the next two sections to obtain a sense of the plausible magnitudes. As I will show, both the data and my model suggest that all three of the following covariances are

[22] Since the typical calibration of the representative-agent model implies a low $M\hat{P}C$, the substitution component is typically dominant in this decomposition, as noticed by Kaplan et al. (2016).

negative:

$$\text{Cov}_I\left(M\hat{P}C_i, URE_i\right) < 0 \quad (21)$$

$$\text{Cov}_I\left(M\hat{P}C_i, NNP_i\right) < 0 \quad (22)$$

$$\text{Cov}_I\left(M\hat{P}C_i, Y_i\right) < 0 \quad (23)$$

suggesting that redistribution *amplifies* the transmission mechanism of monetary policy.

Inequality (21) says that agents with unhedged borrowing requirements have higher marginal propensities to consume than agents with unhedged savings needs. In addition to being supported by the data, in section 5 I will show that it is naturally generated by models with uninsured idiosyncratic risk, with a magnitude that depends on asset durations. Because of this interest rate exposure channel, aggregate consumption is more responsive to real interest rates than measures of intertemporal substitution alone would suggest. In other words, the first-round effect of monetary policy is larger that what the representative-agent model predicts.

Inequality (22) says that net nominal borrowers have higher marginal propensities to consume than net nominal asset holders. This inequality is also both supported by the data and generated endogenously by my model in section 5. It implies that, through its general equilibrium effect on inflation, monetary policy can increase aggregate consumption via a Fisher channel.[23]

Inequality (23) says low-income agents have high MPCs, echoing a finding in much of the empirical literature. On its own, this fact is not enough to sign the earnings heterogeneity channel: we need to know how increases in aggregate income affect agents at different levels of income. More specifically, let

$$\gamma_i \equiv \frac{\partial\left(\frac{Y_i}{Y}-1\right)}{\left(\frac{Y_i}{Y}-1\right)}\frac{Y}{\partial Y} \quad (24)$$

be the elasticity of agent i's relative income to aggregate income. Assume that this is well approximated by a constant γ. Then the earnings heterogeneity channel term in equation (19) simplifies to $\gamma\text{Cov}_I\left(M\hat{P}C_i, Y_i\right)\frac{dY}{Y}$. There is empirical evidence that income risk is countercyclical (for example Storesletten et al. 2004 or Guvenen et al. 2014) and that monetary policy accommodations reduce income inequality (Coibion et al. 2017). These studies all suggest that γ is negative. Combining this fact with

[23] Note that this effect from redistribution is conceptually distinct from the effect of future inflation lowering real interest rates, which has nothing to do with nominal redenomination and is present in representative-agent models with persistent shocks to inflation.

(23), it is likely that monetary expansions increase aggregate consumption because of their endogenous effect on the income distribution.[24]

Independently of the sign of the covariance terms in (19), theorem 3 provides an organizing framework for future research on the role of heterogeneity in the monetary policy transmission mechanism.[25]

3.4 Discussion

I now provide a discussion of my result, highlighting its limitations and possible generalizations.

Interactions between the household and other sectors. The market clearing equations (17) and (18) respectively state that the net nominal positions and the unhedged interest rate exposure of the combined household and government sectors are zero. This is a theoretical restriction that must hold in a closed economy, provided firms are correctly consolidated as part of the household sector. In practice there are two challenges: actual economies are open, and it is difficult to accurately take into account the indirect exposures through firms when measuring NNPs and UREs.

In an open economy, (17) and (18) are no longer true, so price-level and real interest rate changes redistribute between the domestic economy and the rest of the world. For example, Doepke and Schneider (2006) find that the net nominal position of the United States is negative, implying that unexpected inflation redistributes towards the U.S. Given a positive average MPC, consumption should rise by more than what equation (19) predicts. Similarly, Gourinchas and Rey (2007) find that the United States borrows short and lends long on its international portfolio, suggesting that it has a negative unhedged interest rate exposure. Hence, U.S.

[24] Away from separable preferences, an additional *complementarity channel* of monetary policy can arise, even with a representative agent, when preferences are such that increases in hours worked increase the marginal utility of consumption.

[25] An early generation of papers in the heterogeneous agent New Keynesian literature analyzed the transmission of monetary policy under limited heterogeneity. In 'saver-spender' models, such as Bilbiie (2008), 'spender' agents live hand-to-mouth and consume their incomes, so they have $M\hat{P}C = 1$; while 'saver' agents have access to financial markets, with a low $M\hat{P}C$. This has the effect of increasing the aggregate MPC in the economy, raising the importance of income effects relative to substitution effects in equation (19). In 'borrower-saver' models, as in Iacoviello (2005), the high-MPC agents are also borrowers. The literature usually assumes short-term debt, implying (21) and sometimes also nominal debt, implying (22). However, whether (23) holds crucially depends on the assumptions these paper make about the distribution of wages and profits across savers vs spenders.

households benefit *on average* from lower real interest rates, further contributing to the expansionary effects of monetary accommodations on consumption.[26]

The assumption that households and firms are consolidated is also important. For example, the household sector tends to be maturity mismatched, holding relatively short-term assets (deposits) and relatively long-term liabilities (fixed-rate mortgages), but this is to a large extent a counterpart to the reverse situation in the banking sector. In principle, household UREs and NNPs should take into account the indirect exposure to interest rates that each household has through all the firms it has a stake in. In practice this is quite challenging, just as it is challenging to estimate indirect exposures of households to the government balance sheet. Undercounting household exposures to negative-URE sectors will imply a positive $\mathbb{E}_I\left[URE_i\right]$, as in equation (18). However, the logic of theorem 3 shows that this imperfect measurement does not matter to the extent that all marginal rebates from other sectors are immediate and lump-sum. In this sense, the covariance terms provide an important benchmark. In practice, rebates might be delayed, and they might target higher or lower MPC agents, so that the precise numbers may depart from the covariance expression in either direction.

One way to assess the importance of all these effects is to directly measure in the data expressions such as $\mathbb{E}_I\left[M\hat{P}C_i URE_i\right]$ and to compare them to the covariance numbers. These 'no-rebate' numbers replace the covariance terms in (19) under the extreme assumption that none of the outside sectors rebate gains to the household sector. In this context, it is interesting to note the theoretical possibility that the interest rate exposure term $\mathbb{E}_I\left[M\hat{P}C_i URE_i\right]$ may not only be positive, but *larger* than the substitution term in (19). Hence, in a world in which outside rebates are highly delayed or benefit low-MPC agents, real interest rate cuts could *lower* aggregate consumption demand, significantly altering the conventional understanding of how monetary policy operates.[27]

General equilibrium and persistent shocks. Theorem 3 provides the response of consumption to a *transitory* shock to R, P and Y. While this exercise provides an insightful decomposition that has the significant merit of involving measurable sufficient statistics, it has two major limitations.

[26] To the extent that these gains are evenly distributed across the population, these effects can be quantified, respectively, by evaluating $\mathbb{E}_I\left[M\hat{P}C_i\right]\cdot NNP_{US}$ and $\mathbb{E}_I\left[M\hat{P}C_i\right]\cdot URE_{US}$.

[27] This theoretical possibility is sometimes mentioned in economic discussions of monetary policy. See Raghuram Rajan ("Interestingly [...] low rates could even hurt overall spending"), "Money Magic", Project Syndicate, November 11, 2013

First, the exercise is partial equilibrium in nature: in general, theorem 3 does not permit us to solve for the general equilibrium consumption effect of a given exogenous shock. This is because even transitory exogenous shocks tend to have long-lasting effects on agent behavior and the wealth distribution, which in general equilibrium tends to generate adjustments in future interest rates and/or income. Equation (19) does characterize the full equilibrium in my leading case of the benchmark New Keynesian model, but in more general heterogeneous-agent models it will typically only hold as an approximation of the consumption response to a transitory monetary policy shock.[28]

Second, empirically, monetary policy changes tend to be persistent. Persistent shocks make the derivation of sufficient statistics much more difficult: for example, to characterize the effect of future changes in R, one needs to know the distribution of future consumption and income plans.

In the context of a given structural model, a decomposition such as (19) can be performed for any degree of exogenous and endogenous persistence (see section 5.4 and Kaplan et al. 2016).[29] As models grow in complexity and realism, the importance of the channels identified in Theorem 3 can be assessed and refined using such a procedure.[30] I believe that my key finding that redistribution amplifies the effects of monetary policy is likely to remain robust, but it will certainly need to be qualified.

Estimable moments. As discussed above, some of the terms in equation (19) require knowledge of additional information before they can be taken to the data. I make two further assumptions on these structural parameters so as to turn the equation into a full set of estimable moments. For convenience, I also rewrite the decomposition in terms of elasticities.

Corollary 2. *Assume that individuals have common elasticity of intertemporal substitution, $\sigma_i = \sigma$, and common elasticity of relative income to aggregate income,*

[28] For instance, the theorem cannot accommodate capital investment, where a current fall in the real interest rate $dR < 0$ comes together with a future fall in capital income, $d\rho_1 < 0$. A previous working paper version of this paper showed the quality of the approximation $dC \simeq dY$ in the context of a model without investment.

[29] See also Auclert et al. (2017) for an alternative approach that generalizes theorem 3 to shocks with arbitrary persistence and general equilibrium.

[30] One open question in this literature, in particular, is whether the redistribution channel can help account for the empirical impulse response to a monetary policy shock documented in Christiano et al. (2005) and many others.

Table 1: Seven cross-sectional moments that determine consumption in (25)

	Definition	Name	Channel
$\mathcal{E}_R$	$\text{Cov}_I\left(MPC_i, \frac{URE_i}{\mathbb{E}_I[c_i]}\right)$	Redistribution elasticity for R	Interest-rate exposure
$\mathcal{E}_R^{NR}$	$\mathbb{E}_I\left[MPC_i\frac{URE_i}{\mathbb{E}_I[c_i]}\right]$	—, No Rebate	—
$\widehat{S}$	$\mathbb{E}_I\left[(1-MPC_i)\frac{c_i}{\mathbb{E}_I[c_i]}\right]$	Hicksian scaling factor	Substitution
$\mathcal{E}_P$	$\text{Cov}_I\left(MPC_i, \frac{NNP_i}{\mathbb{E}_I[c_i]}\right)$	Redistribution elasticity for P	Fisher
$\mathcal{E}_P^{NR}$	$\mathbb{E}_I\left[MPC_i\frac{NNP_i}{\mathbb{E}_I[c_i]}\right]$	—, No Rebate	—
$\mathcal{M}$	$\mathbb{E}_I\left[MPC_i\frac{Y_i}{\mathbb{E}_I[c_i]}\right]$	Income-weighted *MPC*	Aggregate income
$\mathcal{E}_Y$	$\text{Cov}_I\left(MPC_i, \frac{Y_i}{\mathbb{E}_I[c_i]}\right)$	Redistribution elasticity for Y	Earnings heterogeneity

$\gamma_i = \gamma$ *for all* i. *Then,*

$$\frac{dC}{C} = (\mathcal{M} + \gamma\mathcal{E}_Y)\frac{dY}{Y} - \mathcal{E}_P\frac{dP}{P} + (\mathcal{E}_R - \sigma S)\frac{dR}{R} \tag{25}$$

where $\mathcal{M}$, $\mathcal{E}_Y$, $\mathcal{E}_P$, $\mathcal{E}_R$ *and* S *are all measurable cross-sectional moments summarized in table 1.*

The proof is in appendix A.8. The assumption of a constant γ parametrizes the incidence of increases in aggregate output dY using a convenient functional form.[31] As is clear from equation (24), when $\gamma > 0$, agents with income above the mean benefit disproportionately from such an increase. The opposite happens when $\gamma < 0$. As discussed above, the evidence on the cyclicality of income risk tends to suggest that the latter case is plausible, though a constant γ is obviously a very strong assumption.

Table 1 summarizes the definitions of the moments entering equation (25). I call $\mathcal{E}_P$, $\mathcal{E}_R$ and $\mathcal{E}_Y$ the *redistribution elasticities* of consumption with respect to the price level, the real interest rate and income, since these terms enter explicitly as elasticities in equation (25).[32] The next section measures these numbers in the data.

[31] Such a specification appears, for example, if labor supply is inelastic ($\psi = 0$) and all income is labor income ($d = 0$). In this case, agent i's gross earnings are $e_i Y$, the product of his skills e_i and aggregate output Y. Suppose that the government taxes these earnings at a rate $\tau(Y)$ and rebates them lump-sum. Then post-redistribution earnings are $Y_i = ((1-\tau(Y))e_i + \tau(Y)\mathbb{E}[e_i])Y$. A constant γ_i follows if the net-of-tax rate has constant elasticity with respect to output, i.e. $\frac{\tau'(Y)}{1-\tau(Y)} = -\gamma$.

[32] Calling $\mathcal{E}_Y$ an elasticity is a slight abuse of terminology, since the actual elasticity is $\gamma\mathcal{E}_Y$.

4 Measuring the redistribution elasticities of consumption

This section turns to data from three surveys to get a sense of the empirical magnitudes of each of the terms in table 1. This exercise is not intended as definitive and will need to be refined in future work. Yet it already paints a fairly consistent picture. With these moment estimates in hand, only two parameters in equation (25) remain unknown. σ can be obtained from the vast literature studying the elasticity of intertemporal substitution, and γ can be obtained from studies on the cyclicality of the distribution of income.

4.1 Three surveys, three identification strategies

In order to compute my key cross-sectional moments, I need household-level information on income, consumption, and balance sheets. Several recent household surveys have collected all this information, both in the United States and abroad, with varying degrees of precision. I also need information on $M\hat{P}C$, the marginal propensity out of transitory income shocks.[33] The literature has used various techniques to estimate these MPCs (see Jappelli and Pistaferri 2010 for a survey). Three of the most influential approaches are implementable using public survey data. I compute my moments using all three approaches, each in a different survey. Since I build on standard references in the literature, I restrict myself to a brief description of the methods, and refer the reader to Appendix B and to the original sources for further detail.

My first source of data is the Italian Survey of Household Income and Wealth (SHIW). In 2010, the survey asked households to self-report the part of any hypothetical windfall that they would immediately spend (Jappelli and Pistaferri 2014). The benefit of this approach is that the windfall can be taken as exogenous for all agents, so in principle this empirical measure of MPC is the number that matters for the theory. This approach also provides MPCs at the household level, making it easy to compute covariances with individual balance-sheet information. These are significant advantages, but the numbers only correspond to one specific setting,

[33] Recall that the theory makes a distinction between $M\hat{P}C$, which takes into account the endogenous response of labor supply, and MPC which does not. The methods used to compute MPC either regress observed consumption on observed income, or ask a question to respondents without mentioning a potential labor supply adjustment, so from now on I assume that they measure $M\hat{P}C$, and sometimes write it MPC for convenience.

that of Italy in 2010. Moreover, a concern with self-reported answers to hypothetical situations is that they may not be informative about how households would actually behave in these situations. For these reasons I also turn to other datasets, and to settings where MPCs are estimated from actual behavior.

My second source of data is the U.S. Panel Study of Income Dynamics (PSID) and uses a 'semi-structural' approach to compute MPCs out of transitory income shocks. The procedure is due to Blundell et al. (2008) and has since been popularized by Kaplan et al. (2014) and others in the context of macroeconomics. The idea here is to postulate an income process and a consumption function, and to use restrictions from the theory to back out the MPC out of transitory shocks from the joint cross-sectional distribution of consumption changes and income changes. Since this procedure can only recover an estimate at the group level, I compute my redistribution elasticities by first grouping households into different bins, then estimating MPCs within bins and covariances across bins. One drawback of such a procedure is therefore that it generates significantly larger error bands.

My third and final source of data is the U.S. Consumer Expenditure Survey (CE), in which MPC is identified using exogenous income variation following Johnson et al. (2006). These authors estimate the MPC out of the 2001 tax rebate by exploiting random variation in the timing of the receipt of this rebate across households. As the policy was announced ahead of time, they identify the MPC out of an increase in income that is expected in advance. This is, in general, different from the theoretically-consistent MPC out of an unexpected increase. However, to the extent that borrowing constraints are important, or if households are surprised by the receipt despite its announcement, the estimation gets closer to the MPC that is important for the theory. This procedure also yields an MPC at a group level, so I again estimate covariances across groups.

As discussed in appendix B, each of the three techniques has its own limitations, and no survey contains perfect information on all components of household balance sheets. Notably, the consumption data in the SHIW and the PSID is limited, as are the income and the asset data in the CE. In addition, none of these surveys samples very rich households whose consumption behavior may be an important determinant of aggregate expenditures. Hence, the exercise in this section is tentative and intended to give a sense of magnitudes based on the current state of knowledge in the field. As administrative-quality household surveys become available and more sophisticated identification methods for MPCs arise, a priority for future work is to refine the estimates I provide here.[34]

4.2 Conceptual measurement issues

Even though my analysis is term of elasticities, which are unit-less numbers, the choice of time units is important: MPC needs to be measured over a period of time consistent with the time unit for income, consumption, and maturing elements of the balance sheet. I follow the structure of the datasets, and measure each at an annual rate in the SHIW and the PSID, and at a quarterly rate in the CE.

MPC. As discussed above and in Appendix A.5, my ideal measure of MPC would be one that encompasses both nondurable and durable goods, since this would correspond to the concept that matters for predicting changes in aggregate consumption spending. The question in the SHIW refers to 'spending' without distinguishing between types of purchases, so it is safe to assume that it refers to both durables and nondurables. For my U.S. exercises, I prefer to follow the baseline estimates from Blundell et al. (2008) and Johnson et al. (2006), neither of which include durable goods in MPC estimation. My PSID estimate includes only nondurables, while my CE estimate only includes food. Appendix 4.3 considers robustness to using a broader set of goods into the MPC estimation.

URE. As defined in section 2.2, URE_i measures the total resource flow that a household i needs to invest over the first period of his consumption plan. In each survey, I construct URE_i as

$$URE_i = Y_i - T_i - C_i + A_i - L_i \tag{26}$$

where Y_i is gross income, T_i is taxes net of transfers, C_i is consumption, and A_i and L_i represent, respectively, assets and liabilities that mature over the period over and above the amounts already included in Y_i or C_i. Specifically, C_i includes expenditure on durable goods, rents and interest payments, while Y_i includes income from all sources: labor, dividend, and interest income. Therefore, Y_i comprises the maturing portion of equities and bonds in household portfolios. Moreover, $Y_i - C_i$ includes the

[34] Using administrative Norwegian data and the MPCs of lottery winners, Fagereng et al. (2016) provide estimates of redistribution covariances that are broadly consistent with mine.

'maturing' portion of housing, which I treat as a special asset that pays a dividend equal to its consumption by owner-occupiers.[35] In L_i, I count principal payments on all loans, notably all mortgages, since these are not included in consumption.

The remaining maturing asset and liabilities that I include in A_i and L_i consist of short-term and adjustable-rate assets and liabilities. For these assets, I only observe the stocks, and detailed maturity information is typically absent. I therefore define a benchmark scenario in each survey, and perform an extensive sensitivity analysis in Appendix B.5. I assume in this benchmark that, for every agent i, a) time and savings deposits have a duration of two quarters, b) adjustable-rate mortgages have a duration of three quarters, and c) debt outstanding on credit cards has duration of two quarters. Appendix 4.3 contains detailed information on the asset and liability classes reported in each survey, as well as a comparison across surveys.

NNP and Income. I compute net nominal positions as the difference between directly held nominal assets (mainly deposits and bonds) and directly held nominal liabilities (mainly mortgages and consumer credit). When assets are clearly indicated as shares of a financial intermediary that mostly owns nominal assets (for example, money market mutual funds), I also include the value of these shares in the households' nominal position. However, relative to Doepke and Schneider (2006), I do not calculate the indirect nominal positions arising from holdings of equity or other financial intermediaries, since my data is not sufficiently detailed for this purpose. For my income measure, in keeping with the theory, I use pre-tax income in the PSID and the CE where it is available; in the SHIW I use post-tax income.

Measurement error. Measurement error is a very important issue in this exercise. These errors can stem from many sources: poor data quality, imperfect coverage, underreporting of consumption, or timing differences in the reporting of consumption and income. As discussed in the appendix, each survey has its own strengths and weaknesses. The CE has excellent information on consumption and liabilities, but very poor information on assets. Both the PSID and the SHIW appear to considerably undermeasure consumption. My covariance estimates are unbi-

[35] This differential treatment of housing relative to other durable goods is consistent with the assumption made in national income and product accounts. In the language of section 2.2, the implicit assumption is that the relative price of housing has an elasticity $\epsilon^h = 1$ with respect to the real interest rate, while the relative price of all other durable goods is $\epsilon^d = 0$. I consider robustness to alternative values of ϵ^d in appendix B.5.

Table 2: Main summary statistics from the three surveys

Survey	SHIW		PSID		CE	
Variable	mean	s.d.	mean	s.d.	mean	s.d.
Income after tax ($Y_i - T_i$)	1.31	0.92	2.13	2.63	1.16	1.03
Consumption (C_i)	1.00	0.61	1.00	0.63	1.00	0.83
Maturing assets (A_i)	0.98	2.64	1.46	6.38	0.48	1.70
Maturing liabilities (L_i)	0.34	1.55	0.81	2.11	0.53	1.55
Unhedged interest rate exposure (URE_i)	0.95	3.13	1.78	7.60	0.16	2.36
Nominal assets	0.82	2.61	1.41	5.00	1.90	7.50
Nominal liabilities	0.55	1.65	2.72	3.95	4.97	7.73
Net nominal position (NNP_i)	0.27	2.92	-1.31	6.10	-2.79	10.06
Income before tax (Y_i)	1.31	0.92	2.67	4.11	1.25	1.11
Marginal propensity to consume (MPC_i)	0.47	0.35				
Number of households	7,951		9,620		4,833	

In each survey, 'mean' and 's.d.' represent the sample mean and standard deviation.
All statistics are computed using sample weights.
All variables except for MPC are normalized by average consumption in the sample.

ased provided that the measurement errors in in MPC and its cross-term (URE, NNP or Y) are additive and uncorrelated. Economically, this assumption corresponds to the presence of a 'mismeasurement' sector that rebates gains and losses lump-sum, just as the government does in the setting of theorem 3.[36] Keeping this important caveat in mind, I proceed to my measurement exercise.

4.3 Estimating the redistribution elasticities

Table 2 reports the main summary statistics from each survey, with the appendix containing further detail. Each line is normalized by average consumption in the survey, which facilitates comparability and corresponds to the normalization behind my elasticities in table 1. Note that the average URE is positive all three surveys. One reason, in addition to those highlighted in section 3.4, is that consumption is below income at the mean, especially in the PSID and the SHIW — likely because of underreporting and coverage issues. This is another reason why my preferred estimate of the redistribution elasticity is $\mathcal{E}_R$ rather than $\mathcal{E}_R^{NR}$, which is mechanically pushed up by the high average URE. The average net nominal

[36] For example, by abstracting away from indirect exposures to the banking sector, I tend to overstate the aggregate URE. If gains to the banking sector disproportionately favor low-MPC households, my estimate of the MPC/URE correlation would be biased downwards.

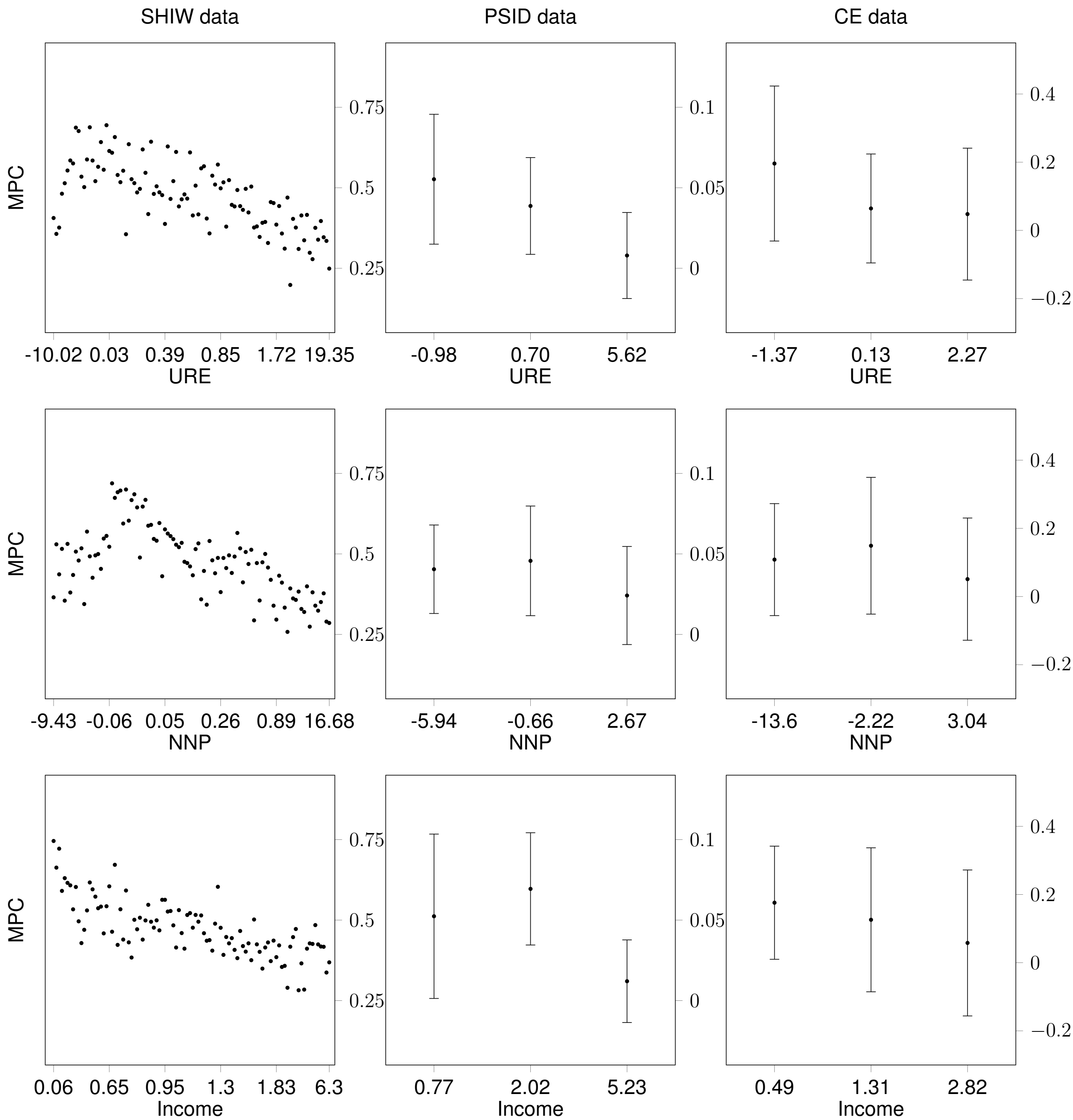

Figure 2: Marginal propensities to consume and the redistribution channels.

position is quite negative in CE and PSID — possibly reflecting poor asset measurement — and moderately positive in the Italian survey, where far fewer households own mortgages.

Figure 2 reports the distribution of MPC by URE, NNP and income across the three surveys. Columns correspond to datasets, and rows to redistribution channels. The first column displays data from the SHIW, where individual MPC information is available. The graphs report the average value of MPC in each percentile of the *x*-axis variable. On the other hand, in the PSID (second column) and the CE (third column), I estimate the MPC by stratifying the population in terciles of the x-axis variable, and then report the point estimate together with confidence intervals within each bin.

Starting with the interest exposure channel, looking across the first row, all three surveys show a negative correlation between MPC and URE. This is particularly apparent in the SHIW data, but the pattern is there in the U.S. surveys as well. A direct implication is that $\mathcal{E}_R < 0$ in each of these datasets: falls in interest rates increase consumption demand via the redistribution channel. Turning to the Fisher channel, we also observe an overall negative correlation, though it is somewhat less pronounced. In particular, MPCs tend to be slightly higher in the center of the NNP distribution than at the extremes, potentially consistent with a 'wealthy hand-to-mouth' explanation as in Kaplan and Violante (2014). This suggests that $\mathcal{E}_P < 0$, consistent with Fisher's hypothesis — unexpected increases in prices tend to increase consumption overall, though this effect is quantitatively small. Finally, across all three surveys, the covariance between MPCs and gross incomes is also negative. Again, this pattern is not entirely clear across the income distribution, consistent with the presence of some wealthy hand-to-mouth individuals. Combined with $\gamma < 0$, a negative $\mathcal{E}_Y$ implies an amplification role for the earnings heterogeneity channel in the transmission of monetary policy.

Moving on to magnitudes, table 3 computes my seven key cross-sectional moments, together with 95% confidence intervals. For the PSID and the CE, the estimation is done across bins by using three bins, just as in figure 2.[37]

Confirming the visual impression from figure 2, the point estimates for the redistribution elasticities $\widehat{\mathcal{E}_R}$, $\widehat{\mathcal{E}_P}$ and $\widehat{\mathcal{E}_Y}$ are negative in all three surveys. However, the magnitudes are relatively small — in particular, the confidence bands in the CE always include zero.[38]

To put these numbers in the context of standard representative-agent analyses, consider that many macroeconomists believe 0.1 to 0.5 as plausible values for the elasticity of intertemporal substitution σ. Equation (25) shows that σ should be compared to $-\mathcal{E}_R/S$ to gauge the relative strength of the redistribution effect. According

[37] Appendix B.5 reports a sensitivity analysis using four to eight bins. The results are little changed.

[38] Moreover, the estimated value of $\mathcal{E}_R^{NR}$ is usually positive, implying that the negative covariance is not strong enough to overwhelm the positive value of URE at the mean. As argued above, taking $\mathcal{E}_R^{NR}$ at face value requires an extreme view of outside-sector rebates.

Table 3: Estimates of table 1's cross-sectional moments using SHIW, CE and PSID

Survey	SHIW		PSID		CE	
	Estimate	95% C.I.	Estimate	95% C.I.	Estimate	95% C.I.
$\widehat{\mathcal{E}_R}$	-0.11	[-0.16,-0.06]	-0.05	[-0.10,-0.00]	-0.09	[-0.26,0.09]
$\widehat{\mathcal{E}_R^{NR}}$	0.34	[0.29,0.39]	0.01	[-0.05,0.06]	-0.05	[-0.23,0.13]
$\widehat{S}$	0.55	[0.53,0.58]	0.97	[0.95,0.98]	0.90	[0.77,1.03]
$\widehat{\mathcal{E}_P}$	-0.07	[-0.12,-0.03]	-0.02	[-0.08,0.04]	-0.11	[-0.83,0.60]
$\widehat{\mathcal{E}_P^{NR}}$	0.05	[0.01,0.10]	-0.07	[-0.13,-0.01]	-0.55	[-1.33,0.23]
$\widehat{\mathcal{M}}$	0.57	[0.55, 0.59]	0.08	[0.03,0.13]	0.14	[-0.12,0.39]
$\widehat{\mathcal{E}_Y}$	-0.05	[-0.07, -0.03]	-0.04	[-0.08,-0.00]	-0.05	[-0.15,0.06]

All statistics are computed using survey weights. In the CE and the PSID, confidence intervals are bootstrapped by resampling households 100 times with replacement.

to the point estimates from table 3, this number is between 0.05 and 0.20. Hence, the data suggests that the redistribution effect might be as important as the substitution effect in explaining why aggregate consumption responds to changes in real interest rates. Similarly, the magnitudes of $\widehat{\mathcal{E}_P}$ and $\widehat{\mathcal{E}_Y}$ are fairly small, so that (unless γ is very negative) neither channel can account on its own for very large movements in consumption. But their combined effect may nevertheless be substantial, and further research is needed to refine the precision of these estimates.

As more sources of joint consumption, income and asset data become available, a better empirical understanding of UREs and NNPs will become possible, helping to shape our understanding of the winners and losers from changes in real interest rates and inflation. Real-time estimates of the redistribution covariances will also provide useful information about the dynamic evolution of the monetary policy transmission mechanism.

4.4 Empirical drivers of the redistribution covariances

While the sufficient statistic approach suggests that only the population-level redistribution elasticities matter to determine an overall effect, in practice it is interesting to understand the empirical drivers of these covariances. For example, is the covariance between MPC and URE negative because older households tend to have lower MPCs and higher UREs? In order to shed light on this and related questions,

Table 4: Covariance decomposition for URE, NNP and income in the SHIW

			$\mathcal{E}_R$		$\mathcal{E}_P$		$\mathcal{E}_Y$	
Z_i	Var (Z_i)	$\widehat{\beta_M}$	$\widehat{\beta_R}$	% expl.	$\widehat{\beta_P}$	% expl.	$\widehat{\beta_Y}$	% expl.
Age bins	0.77	-0.027	0.459	9%	0.521	15%	0.062	3%
Male	0.24	-0.055	0.396	5%	0.285	5%	0.282	7%
Married	0.18	-0.016	0.116	0%	-0.070	-0%	0.417	2%
Years of ed.	18.8	-0.005	0.064	6%	0.031	4%	0.088	17%
Family size	1.71	0.023	-0.107	4%	-0.215	12%	0.122	-10%
Res. South	0.22	0.198	-0.481	19%	-0.255	15%	-0.561	48%
City size	1.21	0.037	0.029	-1%	0.053	-3%	0.068	-6%
Unemployed	0.04	0.189	-0.728	5%	-0.308	3%	-0.624	10%

I perform a covariance decomposition, projecting each covariance onto observable components such as age or education. This procedure is inspired by the law of total covariance: focusing on *URE* for ease of notation, for any covariate Z_i we know that

$$\text{Cov}\left(MPC_i, URE_i\right) = \underbrace{\text{Cov}\left(\mathbb{E}\left[MPC_i|Z_i\right], \mathbb{E}\left[URE_i|Z_i\right]\right)}_{\text{Explained fraction of covariance}} + \underbrace{\mathbb{E}\left[\text{Cov}\left(MPC_i, URE_i|Z_i\right)\right]}_{\text{Unexplained fraction of covariance}} \tag{27}$$

We can then implement this decomposition using an OLS regression, which performs a linear approximation to the conditional expectation function.[39] For any observable covariate Z_i, I run two OLS regressions

$$\begin{aligned} MPC_i &= \alpha_M + \beta_M Z_i + \epsilon_{Mi} \\ URE_i &= \alpha_R + \beta_R Z_i + \epsilon_{Ri} \end{aligned}$$

and compute the covariance between the fitted values $\widehat{MPC_i}$ and $\widehat{URE_i}$ to get an empirical counterpart of the explained component in (27). This gives me the part of the covariance that can be explained by Z_i, since

$$\begin{aligned} \text{Cov}\left(MPC_i, URE_i\right) &= \text{Cov}\left(\widehat{MPC_i} + \widehat{\epsilon_{Mi}}, \widehat{URE_i} + \widehat{\epsilon_{Ri}}\right) \\ &= \text{Cov}\left(\widehat{\beta_M} Z_i + \widehat{\epsilon_{Mi}}, \widehat{\beta_R} Z_i + \widehat{\epsilon_{Ri}}\right) \\ &= \text{Var}\left(Z_i\right) \widehat{\beta_M}\widehat{\beta_R} + \text{Cov}\left(\widehat{\epsilon_{Mi}}, \widehat{\epsilon_{Ri}}\right) \end{aligned} \tag{28}$$

where the last line follows because, by construction, $\text{Cov}\left(\widehat{\epsilon_{Mi}}, Z_i\right) = \text{Cov}\left(\widehat{\epsilon_{Ri}}, Z_i\right) = 0$. For example, in table 4, when Z_i is age, $\widehat{\beta_M}$ is negative and $\widehat{\beta_R}$ is positive, so older agents do tend to have lower *MPC* and higher *URE*. However, on its own, age can only explain 9% of the total covariance.

[39] This is similar to implementing the law of total variance using R^2.

This procedure is straightforward to implement in the SHIW, where MPC is available at the individual level. Table 4 reports these results using as control variables all those that Jappelli and Pistaferri (2014) use to explain *MPC*, one covariate at a time. For each of my three redistributive channels, I report each of the terms in the decomposition (28), as well as the fraction of the variance explained. In Appendix B.6, I generalize this approach to multiple covariates, and also report graphs of *URE* and *NNP* by age and income bins in each survey. All of these tend to give a consistent message: age, education and income are all negatively correlated with MPC and positively correlated with URE and NNP, so they help explain the negative covariance overall.

5 Sufficient statistics in a Huggett model

This section puts some additional structure to the model of section 3 to connect the empirical magnitudes estimated in the previous section back to theory. It answers the following four main questions: 1) Can a simple model rationalize the empirical signs and magnitudes obtained in the previous section? 2) What are some key theoretical determinants of these redistribution elasticities? 3) How robust are the sufficient statistics predictions to large shocks? 4) What about persistent shocks such as those likely to prevail in practice?

Since I explicitly specify the heterogeneity and the driving processes, the sufficient statistics now become endogenous. However, I do not explicitly model the endogenous determination of incomes, and instead assume an endowment economy with exogenous labor supply. Endogenizing the earnings heterogeneity channel remains a significant challenge for the burgeoning literature on New Keynesian Heterogeneous-Agent models, so I do not attempt to do this here, and provide a discussion at the end of the section.

5.1 Environment

The economy is now populated by a continuum of infinitely-lived, ex-ante identical but ex-post heterogenous households indexed by $i \in [0,1]$. Agents do not work, but face face idiosyncratic uncertainty with respect to their endowment of goods $\{y_{it}\}$ and their discount factor $\{\beta_{it}\}$. The process for the exogenous idiosyncratic state $\mathrm{s}_{it} = (y_{it}, \beta_{it})$ is uncorrelated across agents and follows a Markov chain $\Gamma(\mathrm{s}'|\mathrm{s})$ over time. This Markov chain is assumed to have a stationary distribution $\varphi(\mathbf{s})$, which I take to be the cross-sectional distribution of idiosyncratic states at $t = 0$. There is no aggregate uncertainty: the path for all macroeconomic variables is perfectly anticipated.

Labor supply is exogenous so all households value consumption streams only. They do so with separable preferences, as in (8), with inelastic labor supply ($\psi = 0$) and common elasticity of intertemporal σ.

I assume that there are two assets available for trade, both risk-free, long-term bonds with identical rates of decay δ as in section 2.3. One of these assets is a nominal asset and one is a real asset. Prices are expected to remain constant forever, and therefore households are completely indifferent between both types of bonds. To break indifference, I assume that each household allocates a fraction κ of his portfolio to the nominal asset. A borrowing constraint limits the size of bond issuances so that the market value of real end-of-period liabilities is bounded by a limit $\overline{D}$, which takes the form in (10).

Readers will recognize the standard incomplete markets model, taken here in partial equilibrium as in Deaton (1991), Carroll (1997), and which forms the basis of general equilibrium variants such as Bewley (1980), Huggett (1993) and Aiyagari (1994). The only difference is that assets may be nominal and have long maturity, two features that are crucial characteristics of household balance sheets. I have abstracted away from many additional important features in household finance, such as portfolio choice, to focus on the key determinants of sufficient statistics.

In this environment, Theorem 1 applies to every individual agent, with $\hat{MPC} = MPC$ since labor supply does not enter preferences. I consider a calibration of the steady-state of this model in which aggregate income and consumption are equal ($C = \mathbb{E}_I[y_i]$). This can be interpreted as the general equilibrium a closed economy with no government spending or taxes. Starting from such a steady state, Theorem 3 applies, allowing me to ask my four main questions of this section.

5.2 Steady-state calibration and solution method

I perform my calibration at quarterly frequency. I assume a steady state real interest rate of 3% and a household debt to consumption ratio 113% — the U.S. level

Table 5: Calibration parameters, targets, and main sufficient statistics

Parameters		Value	Targets	
Elasticity of intertemporal substitution	σ	0.5		
Impatient discount factor	β^I	0.93	Average MPC	0.25
Patient discount factor	β^P	0.99	Real interest rate (annual)	3%
Borrowing limit (% of per capita annual C)	$\overline{D}$	195%	Household debt (% of C)	113%
Outcomes				
Redistribution elasticity for R ($\delta = 0.95$)	$\mathcal{E}_R$	-0.09	See Figure 3a	
Hicksian scaling factor	S	0.84		
Redistribution elasticity for P ($\kappa = 0$)	$\mathcal{E}_P$	-1.8	See Figure 3b	
Income-weighted MPC	$\mathcal{M}$	0.17		
Redistribution elasticity for Y	$\mathcal{E}_Y$	-0.08		

for 2013. Since there is no net savings, total assets are also 113% of consumption, which is consistent with data on interest-paying assets held by the household sector.[40] As already noted, I assume that there is no inflation at steady-state ($\Pi = 1$), and consider a range of calibrations for bond durations, from 1 quarter to 10 years ($\delta \in [0, 0.95]$), and for inflation indexation ($\kappa \in [0, 1]$).[41] As a benchmark, Doepke and Schneider (2006) report that the average duration of U.S. household assets and liabilities is 4.5 years (see their figure 3), which falls in the middle of my range. On the other hand, the household assets that my calibration includes are entirely nominal, making $\kappa = 0$ a useful reference point.

I follow the vast majority of the literature in postulating an income process that follows an $AR(1)$ process in logs at quarterly frequency, and follow Guerrieri and Lorenzoni (2015) to calibrate this process.[42] I normalize $\overline{y}$ such that $\mathbb{E}[y] = 1$.

Since the moments of the redistribution channel all feature a prominent role for MPCs, I make sure that my model generates *average* marginal propensities to consume that are in line with the empirical evidence. The empirical literature replicated above, and reviewed in more detail in Jappelli and Pistaferri (2010), consistently finds numbers between 0.1 to 0.4 at an annual rate. I settle for a number in the middle of this range, and target an average quarterly marginal propensity to con-

[40] According to the U.S. Financial Accounts, in 2013 households held interest-paying liabilities worth \$13trn and interest-paying assets worth \$12.8trn. I define the former as the sum of mortgages and consumer credit, the latter as time and savings deposits and credit market instruments.

[41] The duration of the nominal bond is $\frac{R\Pi}{R\Pi - \delta} = \mathcal{D}$, so $\frac{\delta}{\Pi} = R\left(1 - \frac{1}{\mathcal{D}}\right)$. $\delta = 0$ and $\mathcal{D} = 1$ for short-term debt.

[42] Specifically, the process is $\log y_t^i - \log \overline{y} = \rho_y \left(\log y_{t-1}^i - \log \overline{y}\right) + \sigma_y \sqrt{1 - \rho^2} \epsilon_t^i$ with $\epsilon_t^i \sim \mathcal{N}(0, 1)$, with a coefficient of mean reversion of $\rho_y = 0.96$ at quarterly frequency, and a cross-sectional standard deviation of log income of $\sigma_y = 0.52$. I discretize the process using a ten state Markov chain.

sume of 0.25. It is well-known that the benchmark incomplete markets model cannot generate this level of average MPCs (see for example Kaplan and Violante 2014). I follow the simple Krusell and Smith (1998)–Carroll et al. 2017 solution of assuming slow-moving time preference heterogeneity, with agents alternating between patience (discount factor β^P) and impatience (discount factor β^I).[43] I then jointly calibrate β^P, β^I and the borrowing limit $\overline{D}$ to achieve my targets for the average MPC, household debt, and a closed current account at $R = 3\%$.[44] The top row of table 5 summarizes my benchmark parameters.

The model is solved using a version of Carroll (2006)'s endogenous gridpoints methods. Details are provided in appendix C.

5.3 Sufficient statistics in the model and the data

The bottom row of table 5 shows the five key endogenous sufficient statistics in the model. The Hicksian scaling factor S and the income-weighted MPC $\mathcal{M}$ take on values consistent with the empirical evidence. $\mathcal{M}$ is below the calibrated average MPC because of the negative covariance between MPCs and income, which is given by $\mathcal{E}_Y = -0.08$. The fact that this covariance is both negative and small is consistent with the empirical evidence from table 3, and is one success of the model. It results from the fact that, in the model, MPCs are strongly negatively correlated with liquid wealth ('cash-on-hand'), but income and cash on hand are not that highly correlated precisely because liquid wealth is used to smoothe income fluctuations.

Next, when I choose $\delta = 0.95$ to match an average asset duration of 4.5 years, I obtain a redistribution elasticity for real interest rates, $\mathcal{E}_R = -0.08$, that is negative but small, as it is in the data. This is because long durations imply endogenously small unhedged interest rate exposures, since households roll over only a fraction of their wealth every quarter. As a consequence, falls in real interest rates imply a limited amount of redistribution — though this redistribution does favor high-MPC households on average. However, the left panel of figure 3 shows that this result is very sensitive to the assumed duration of assets. Typical calibrations of Bewley

[43] My Markov process is such that the stationary population distribution contains patient and impatient agents in equal numbers, and that consumers stay in their patience state for 50 years on average.
[44] Intuitively, β^P controls net asset accumulation, β^I the average MPC and $\overline{D}$ the debt-to-GDP ratio, so a global solver has no difficulty finding a solution to this system of three equations in three unknowns.

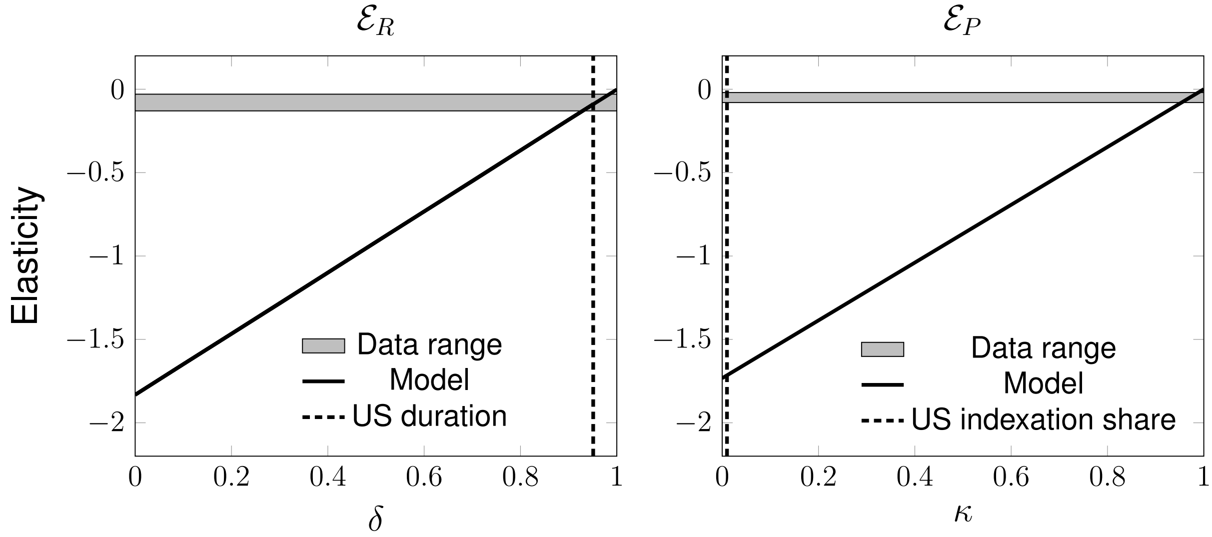

Figure 3: Sufficient statistics: model vs data

models assume that debt is short-term ($\delta = 0$). As the figure shows, assuming short-term debt implies a very large and negative $\mathcal{E}_R$. In such a calibration, the redistributive effects of real interest rate changes completely swamp the substitution effects: $-\mathcal{E}_R/S$ is more than four times larger than the elasticity of intertemporal substitution σ, implying that more than 80% of the consumption effects of real interest rate changes come from redistribution. This finding implies a very important role for the maturity structure in determining the aggregate effects of monetary policy changes.

There are two equivalent ways of interpreting the more muted response of the economy to monetary policy shocks under longer asset durations. The first is that long durations reduce the endogenous amount of unhedged interest rate exposures — making everyone's consumption less sensitive to changes in real interest rates. A second and more subtle interpretation is that under longer asset maturities, expansionary monetary policy creates more capital gains for asset holders and additional upward revaluation of liabilities for borrowers. These capital gains and losses redistribute against the economy's MPC gradient, and therefore make monetary policy less potent in affecting consumption.

Such a role for the maturity structure in monetary policy transmission is consistent with the cross-country structural VAR evidence presented in Calza et al. (2013). It suggests that wealth redistribution is the primary reason why monetary policy affects consumption in a country like the United Kingdom, where mortgages have adjustable rates.

Turning to the redistributive role of inflation, the model with purely nominal assets ($\kappa = 0$) implies a counterfactually large elasticity of aggregate consumption with respect to increases in the price level, $\mathcal{E}_P = -1.77$. In contrast to the data, this version of the model implies instead very powerful redistribution through the Fisher channel. There is a strong intuition for this result. Inflation redistributes along the asset dimension, which in this class of models is highly correlated with MPC (a consequence of the concavity of the consumption function). If all assets are nominal, inflation directly affects agents' real asset positions. Of course, as the right panel of figure 3 shows, more inflation indexation brings the model closer to the empirical results. However, matching the data requires assuming at least 90% inflation indexation, which is clearly counterfactual. Understanding this discrepancy between model and data, and evaluating more carefully the role of the Fisher channel in monetary policy transmission, is a priority for future research.

Whether as cross-checks or as direct targets for calibration, sufficient statistics play a promising role in disciplining heterogeneous-agent general equilibrium models going forward.

5.4 Effects of shock size and persistence

I now explore the role of persistence, focusing on real interest rate changes. Specifically, in this exercise, I maintain a constant price level $P_t = \overline{P}$ and change the real interest rate by

$$R_t - R^* = \rho_R \left(R_{t-1} - R^*\right) - \epsilon_t, \quad t \geq 1 \tag{29}$$

This allows me to answer my two final questions: how robust are the predictions of Theorem 1 for large transitory shocks? And do the main intuitions regarding the role of UREs survive with persistent shocks?

Asymmetric effects from increases and cuts. My first exercise maintains transitory shocks ($\rho_R = 0$) but varies the size of ϵ_0. The left panel of figure 4 shows the result of this exercise as a function of ϵ_0, for increases and cuts of up to 500bps. Two conclusions emerge. First, in my benchmark calibration with 4.5 year dura-

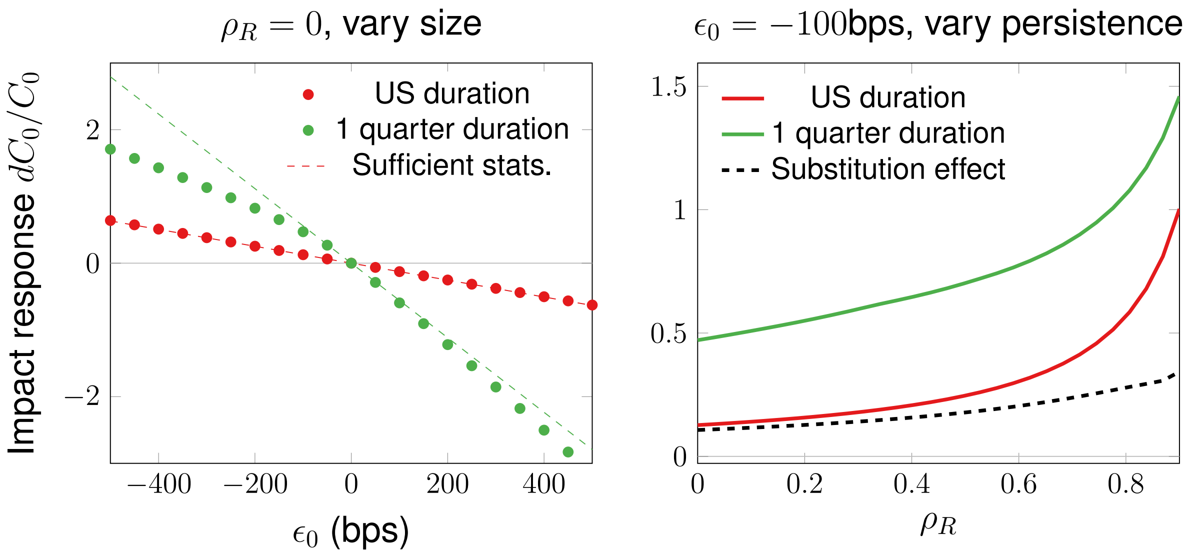

Figure 4: Effect of shock persistence and shock size

tions, the sufficient statistic approximation is excellent in both directions, including for large increases and cuts. Second, in the economy with one quarter duration, where the elasticity is much larger, an asymmetry emerges between the effects of large increases and large cuts: large enough *cuts* in interest rates do not stimulate consumption as much as the sufficient statistics predicts. This asymmetric effect can be traced back to the asymmetric behavior of borrowing-constrained agents to increases and falls in *income*. While these agents have to cut consumption one for one in response to income falls, their MPC out of moderate increases is below 0.3. Because their debt is short term in the ARM calibration, falls in interest rates effectively act as reductions in payments on their credit limit, and therefore as increases in income. In the aggregate, this generates an effective reduction in MPC differences that is strong enough to affect the quantitative magnitude of the redistribution channel. Increases in interest rates do not have the same feature, since the MPC of borrowers out of increases in interest payments is exactly one, as captured by the sufficient statistics.

This type of asymmetric effect of monetary policy has received empirical support (see for example Cover 1992; de Long and Summers 1988 and recently Tenreyro and Thwaites 2016). My explanation, which has to do with asymmetric MPC differences in response to policy rate changes, provides an alternative to the traditional Keynesian interpretation of this fact, which relies on downward nominal wage rigidities.[45]

Robust predictions from persistent shocks. The right panel of figure 4 displays the impact response of the economy as a function of ρ_R, for a shock to ϵ_0 of 100 basis points, under a short duration calibration and a long duration calibration (the degree of indexation of contracts is of course irrelevant). The graph decomposes the response as the sum of an income effect and a substitution effect. Consider first the output effect from a transitory shock ($\rho_R = 0$, to the left of the graph). As we already know, the benchmark calibration has a limited role for the redistribution effect relative to the substitution effect, whereas the redistribution effect is much more important in the economy with short durations. The key message of this graph is that this pattern continues to hold no matter what the persistence of the shock ρ_R is. As shock persistence grows, the substitution effect grows but the redistribution effect grows as well. However, the graph shows that it is not quite right to hold the relative sizes of these two effects fixed: in fact, in the model, redistribution becomes more important as the shock becomes more persistent.

This result shows both the benefits and the costs of using the sufficient statistic approach. Measured sufficient statistics for transitory shocks can be informative about what would happen under more persistent shocks, at least in terms of direction. Yet they are not structural objects, so they cannot be used as elasticities that stay constant as persistence changes.

5.5 General equilibrium effects on income

The model presented above takes incomes as exogenous. This distinguishes it from the recent wave of Heterogeneous-Agent New Keynesian models which endogenize aggregate income and its distribution. My analysis shows that the endogenous distribution of income can matter a great deal for monetary policy transmission because of the earnings heterogeneity channel. As illustrated in Appendix A.1, models with sticky prices tend to generate procyclical wages, procyclical capital

[45] While my U.S. benchmark calibration does not feature asymmetric effects of interest rates, in practice, the refinancing option embedded in fixed rate mortgages in the United States is likely to create an asymmetric effect in the opposite direction from the one I stress here. See Wong (2015) for theory and empirical evidence along these lines.

income and countercyclical profits. On the contrary, models with sticky wages tend to generate countercyclical wages and procyclical profits. Hence both the nature of nominal rigidities and the way in which labor, capital and profits are distributed across the population matters for the results. A successful model needs to match the empirical evidence on the cyclicality of the distribution of income by income type. Models such as those of Gornemann et al. (2012) and Kaplan et al. (2016) make progress along these lines.

6 Conclusion

This paper contributes to our understanding of the role of heterogeneity in the transmission mechanism of monetary policy. I identified three important dimensions along which monetary policy redistributes income and wealth, and argued that each of these dimensions was likely to be a source of aggregate effects on consumption. My classification holds in many environments and provides a simple, reduced-form approach to computing aggregate magnitudes. Hence it can guide future work on the topic, both theoretical and empirical.

An important finding of my paper is that capital gains and losses, both nominal and real, matter for understanding monetary policy transmission. This finding has broad implications for monetary policy. A change in the inflation target can create large redistribution in favor of high MPC agents and be expansionary over and beyond its effect on real interest rates. With long asset maturities, lower real interest rates can benefit asset holders with lower MPCs and make interest rate cuts less effective at increasing aggregate demand than they would otherwise be. Monetary policy becomes intertwined with fiscal policy, but also with government debt maturity management and mortgage design policies.

These are just some of the macroeconomic consequences of the presence of large and heterogeneous marginal propensities to consume, which are a robust feature of household micro data. My investigation was very much a first pass, and opens up many avenues for future research on monetary policy with heterogeneous agents.

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Appendix

A Proofs for sections 2 and 3

A.1 The standard New Keynesian model

This section shows that, in the standard New Keynesian model with sticky Calvo prices, the impulse response to the path for prices P_t, real discount rates q_t, real wages w_t and unearned income are those given by my main experiment in figure 1. I only outline the elements of the model relevant to my argument, the reader is referred to the textbook treatments of Woodford (2003) or Galí (2008) for details.

I consider the model in its 'cashless limit', with no aggregate uncertainty. The model features a representative agent with separable utility trading in one-period nominal bonds and holding a fixed stock of capital k, so equation (1) simplifies to

$$\sum \beta^t \{u(c_t) - v(n_t)\}$$
$$P_t c_t + ({}_t Q_{t+1}) B_{t+1} = P_t \pi_t + W_t n_t + B_t + P_t \rho_t k$$

Here ρ_t denotes the real rental rate of capital, so $\rho_t k$ are total real rents, and π_t are real firm profits. Together, rents and profits make up the unearned income in this economy. Consumption c_t is an aggregate of intermediate goods, with constant elasticity of substitution ϵ. Hence the price index, aggregating the individual goods prices p_{jt}, is $P_t = \left(\int_0^1 p_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$.

Each good j is produced under monopolistic competition with constant returns to scale and unit productivity. The production function is

$$y_{jt} = F(k_{jt}, l_{jt}) = k_{jt}^{\alpha} l_{jt}^{1-\alpha}$$

Firms can only adjust their price with probability θ each period, independent across firms and periods (the Calvo assumption). Nominal wages W_t and nominal rents are flexible. Cost minimization by the firm therefore implies

$$\begin{aligned} \rho_t P_t &= \Lambda_{jt} F_k(k_{jt}, l_{jt}) \\ W_t &= \Lambda_{jt} F_l(k_{jt}, l_{jt}) \end{aligned}$$

for some scalar Λ_{jt} representing the nominal marginal cost of production for firm j. Hence

$$\frac{F_k\left(k_{jt}, l_{jt}\right)}{F_l\left(k_{jt}, l_{jt}\right)} = \frac{F_k\left(\frac{k_{jt}}{l_{jt}}, 1\right)}{F_l\left(\frac{k_{jt}}{l_{jt}}, 1\right)} = \frac{\rho_t}{w_t}$$

so all firms have the same capital-labor ratio $\frac{k_{jt}}{l_{jt}} = \frac{k_t}{l_t}$, and hence all firms have the same nominal marginal cost of production Λ_t.

As is well-known, a first-order approximation to the equilibrium equations of this model is given by the system of three equations

$$\log\left(\frac{c_t}{\overline{c}}\right) = \log\left(\frac{c_{t+1}}{\overline{c}}\right) - \sigma\left(i_t - \log\left(\frac{P_{t+1}}{P_t}\right) - \varrho\right) \tag{A.1}$$

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \beta\log\left(\frac{P_{t+1}}{P_t}\right) + \kappa\log\left(\frac{c_t}{\overline{c}}\right) \tag{A.2}$$

$$i_t = \varrho + \phi_\pi \log\left(\frac{P_t}{P_{t-1}}\right) + \epsilon_t \tag{A.3}$$

where $\overline{c}$ is the level of consumption that would prevail under flexible prices, which (normalizing $k = 1$) solves

$$\frac{v'\left((\overline{c})^{\frac{1}{1-\alpha}}\right)}{u'\left(\overline{c}\right)} = \frac{\epsilon - 1}{\epsilon}\frac{(1-\alpha)}{\overline{c}} \equiv \overline{w}$$

$\varrho = \beta^{-1} - 1$ is the steady-state net real interest rate, $\sigma = -\frac{u'(\overline{c})}{cu''(\overline{c})}$ is the elasticity of substitution around $\overline{c}$, and κ is the slope of the Phillips curve (a function of model parameters). Equation (A.3) is a Taylor rule describing the behavior of monetary policy. We assume that $\phi_\pi > 1$, which guarantees equilibrium uniqueness. We consider the effects of a time-0 monetary policy loosening, $\epsilon_0 < 0$ and $\epsilon_t = 0$ for $t \geq 1$, assuming the system was at steady-state at $t = -1$, with constant price level $\overline{P}$.

It is easy to guess and verify that the equilibrium features $i_t = \rho$, $P_t = P_{t-1}$ and $c_t = \overline{c}$ for $t \geq 1$. Solving backwards, this implies that

$$i_0 = \rho + \frac{1}{1 + \kappa\sigma\phi_\pi}\epsilon_0$$

$$\log\left(\frac{c_0}{\overline{c}}\right) = -\frac{\sigma}{1 + \kappa\sigma\phi_\pi}\epsilon_0$$

$$\log\left(\frac{P_0}{\overline{P}}\right) = -\frac{\kappa\sigma}{1 + \kappa\sigma\phi_\pi}\epsilon_0$$

In other words, a monetary loosening raises c_t at $t = 0$ only, and raises P_t immediately and permanently. (Firms that get an opportunity to reset at $t = 0$ all increase their price above $\overline{P}$, pulling up the price level to P_0. Thereafter, all firms that get a chance reset their price to P_0, so there is no inflation.) To a first-order approximation, the real wage satisfies

$$w_t = \frac{v'\left(c_t^{\frac{1}{1-\alpha}}\right)}{u'\left(c_t\right)}$$

so w_t increases at $t=0$ only and then reverts to $\overline{w}$. Moreover, real rents are

$$\rho_t = \frac{\alpha}{1-\alpha} w_t c_t^{\frac{1}{1-\alpha}}$$

so they also increase at $t=0$ and then revert to $\overline{\rho} = \frac{\alpha}{1-\alpha}\overline{w}\left(\overline{c}\right)^{\frac{1}{1-\alpha}}$.[46] Date-0 nominal and real state prices are $Q_0 = q_0 = 1$ and, for $t \geq 1$, given that $P_t = P_0$,

$$q_t = Q_t = \prod_{s=0}^{t-1}\left({}_sQ_t\right) = \frac{1}{1+i_0}\beta^{t-1}$$

Hence, the path of q_t and Q_t for $t \geq 1$ is shifted upwards by $\frac{dq_t}{q_t} = \frac{dQ_t}{Q_t} = -\frac{dR}{R}$ where the proportional real interest rate change is $\frac{dR}{R} = \frac{d\epsilon_0}{(1+\kappa\sigma\phi_\pi)}\frac{1}{(1+\rho)}$. Finally, aggregate profits are, to first-order, given by

$$\pi_t = c_t - w_t n_t - \rho_t k = c_t\left(1 - \frac{1}{1-\alpha}\frac{v'\left(\left(c_t\right)^{\frac{1}{1-\alpha}}\right)}{u'\left(c_t\right)}c_t\right) \tag{A.4}$$

Hence they also deviate only at $t=0$ from their steady state value of $\overline{d} = \frac{\overline{c}}{\epsilon}$. The first term in (A.4) is volume, which rises with c_0. The second term is the markup, which falls with c_0. In typical calibrations, the markup effect dominates and profits fall in response to an expansionary monetary shock $\epsilon_0 < 0$.

Collecting results, the timing of changes for w_t, P_t and q_t, as well as unearned income $\rho_t k + \pi_t$, is exactly that depicted in figure 1, as claimed in the main text.

A.2 Proof of theorem 1

The proof is greatly simplified by first applying a simple renormalization of discount factors. Instead of the present value normalization $q_0 = 1$, I normalize $q_1 = 1$ and let q_0 vary. Then, setting

$$\frac{dq_0}{q_0} = \frac{dR}{R} \tag{A.5}$$

yields the experiment in figure 1. Intuitively, a rise in the relative price of future goods relative to a current good is the same as a fall in the price of that current

[46] Since price dispersion rises as a result of the monetary policy shock, the nonlinear solution features a real wage that is different from steady state even beyond $t \geq 1$, but the difference is second order in ϵ_0.

good relative to all future goods. This renormalization is innocuous since there is a degree of freedom in choosing discount factors.

Given the experiment, we can hold q_t fixed for $t \geq 1$. Hence, only three parameters y_0, w_0 and q_0 vary, together with the sequence $\{P_t\}$.

With this renormalization, the proof has three steps: first, I apply Slutky's theorem to break down dc and dn into income and substitution effects. Second, I work out explicit expressions for MPC and MPN. Finally, I calculate compensated derivatives, and use my expressions from the second step to simplify their expressions.

Step 1: Slutky's theorem. Recall that the sequences $\{q_t\}$ and $\{w_t\}$ are fixed in the experiment, except for q_0 and w_0. Define the following expenditure function

$$e\left(q_0, w_0, U\right) = \min\left\{\sum_t q_t\left(c_t - w_t n_t\right) \quad \text{s.t.} \quad \sum_t \beta^t\left\{u\left(c_t\right) - v\left(n_t\right)\right\} \geq U\right\} \tag{A.6}$$

and let c_0^h, n_0^h be the resulting compensated (Hicksian) demands for time-0 consumption and hours. Applying the envelope theorem, we obtain a version of Shephard's lemma:

$$e_{q_0} = c_0 - w_0 n_0 \tag{A.7}$$

$$e_{w_0} = -q_0 n_0 \tag{A.8}$$

Define 'unearned' wealth as

$$\widetilde{\omega} \equiv \sum_{t\geq 0} q_t\left(y_t + \left({}_{-1}b_t\right) + \left(\frac{{}_{-1}B_t}{P_t}\right)\right)$$

and note that, given the variation we consider,

$$d\widetilde{\omega} = \left(y_0 + \left({}_{-1}b_0\right) + \left(\frac{{}_{-1}B_0}{P_0}\right)\right) dq_0 + q_0 dy_0 - \sum_{t\geq 0} q_t\left(\frac{{}_{-1}B_t}{P_t}\right)\frac{dP_t}{P_t} \tag{A.9}$$

Using the Fisher equation $\frac{q_t}{P_t} = \frac{Q_t}{P_0}$, and the fact that $\frac{dP_t}{P_t} = \frac{dP}{P}$ is a constant, the last term rewrites

$$\sum_{t\geq 0} q_t\left(\frac{{}_{-1}B_t}{P_t}\right)\frac{dP_t}{P_t} = \sum_{t\geq 0} Q_t\left(\frac{{}_{-1}B_t}{P_0}\right)\frac{dP}{P} = q_0 NNP\frac{dP}{P}$$

where we have defined the household's net nominal position as the present value of his nominal assets

$$q_0 NNP \equiv \sum_{t\geq 0} Q_t\left(\frac{{}_{-1}B_t}{P_0}\right)$$

Moreover, defining

$$URE \equiv w_0 n_0 + y_0 + ({}_{-1}b_0) + \left(\frac{{}_{-1}B_0}{P_0}\right) - c_0$$

we can rewrite (A.9) as

$$d\widetilde{\omega} = (URE + c_0 - w_0 n_0)\, dq_0 + q_0 dy_0 - q_0 NNP \frac{dP}{P} \tag{A.10}$$

Next, define the indirect utility function that attains $\widetilde{\omega}$ as

$$V(q_0, w_0, \widetilde{\omega}) = \max \left\{ \sum_t \beta^t \{u(c_t) - v(n_t)\} \quad \text{s.t.} \quad \sum_t q_t (c_t - w_t n_t) = \widetilde{\omega} \right\} \tag{A.11}$$

Let c_0, n_0 denote the resulting Marshallian demands. Applying the envelope theorem, we find

$$\frac{\partial V}{\partial q_0} = -\frac{u'(c_0)}{q_0}(c_0 - w_0 n_0) \tag{A.12}$$

$$\frac{\partial V}{\partial w_0} = \frac{u'(c_0)}{q_0} q_0 n_0 \tag{A.13}$$

$$\frac{\partial V}{\partial \widetilde{\omega}} = \frac{u'(c_0)}{q_0} \tag{A.14}$$

As in the proof of Slutky's theorem, we next differentiate along the identities

$$\begin{aligned} c_0^h(q_0, w_0, U) &= c_0(q_0, w_0, e(q_0, w_0, U)) \\ n_0^h(q_0, w_0, U) &= n_0(q_0, w_0, e(q_0, w_0, U)) \end{aligned}$$

to find that Marshallian and Hickisan derivatives are related via

$$\frac{\partial c_0^h}{\partial q_0} = \frac{\partial c_0}{\partial q_0} + \frac{\partial c_0}{\partial \widetilde{\omega}} e_{q_0} \qquad \frac{\partial c_0^h}{\partial w_0} = \frac{\partial c_0}{\partial w_0} + \frac{\partial c_0}{\partial \widetilde{\omega}} e_{w_0} \tag{A.15}$$

$$\frac{\partial n_0^h}{\partial q_0} = \frac{\partial n_0}{\partial q_0} + \frac{\partial n_0}{\partial \widetilde{\omega}} e_{q_0} \qquad \frac{\partial n_0^h}{\partial w_0} = \frac{\partial n_0}{\partial w_0} + \frac{\partial n_0}{\partial \widetilde{\omega}} e_{w_0} \tag{A.16}$$

Next, define

$$MPC \equiv q_0 \frac{\partial c_0}{\partial \widetilde{\omega}} \tag{A.17}$$

$$MPN \equiv q_0 \frac{\partial n_0}{\partial \widetilde{\omega}} \tag{A.18}$$

these express the dollar-for-dollar (or hour-for-dollar) marginal propensities to consume and work at date 0: indeed,

$$\frac{\partial c_0}{\partial y_0} = \frac{\partial c_0}{\partial \widetilde{\omega}} \frac{\partial \widetilde{\omega}}{\partial y_0} = \frac{MPC}{q_0} q_0 = MPC$$

and similarly $\frac{\partial n_0}{\partial y_0} = MPN$.

Totally differentiating the Marshallian consumption function and using (A.10), we find

$$dc_0 = \frac{\partial c_0}{\partial q_0}dq_0 + \frac{\partial c_0}{\partial w_0}dw_0 + \frac{\partial c_0}{\partial \widetilde{\omega}}\left((URE + c_0 - w_0 n_0)\, dq_0 + q_0 dy_0 - q_0 NNP\frac{dP}{P}\right)$$

Using (A.15)–(A.16),

$$\begin{aligned} dc_0 &= \left(\frac{\partial c_0^h}{\partial q_0} - \frac{\partial c_0}{\partial \widetilde{\omega}}e_{q_0}\right)dq_0 + \left(\frac{\partial c_0^h}{\partial w_0} - \frac{\partial c_0}{\partial \widetilde{\omega}}e_{w_0}\right)dw_0 \\ &\quad + \frac{\partial c_0}{\partial \widetilde{\omega}}\left((URE + c_0 - w_0 n_0)\, dq_0 + q_0 dy_0 - q_0 NNP\frac{dP}{P}\right) \\ &= \frac{\partial c_0}{\partial \widetilde{\omega}}\left(-e_{w_0}dw_0 + q_0 dy_0 + (-e_{q_0} + URE + c_0 - w_0 n_0)\, dq_0 - NNP\frac{dP}{P}\right) \\ &\quad + \frac{\partial c_0^h}{\partial q_0}dq_0 + \frac{\partial c_0^h}{\partial w_0}dw_0 \end{aligned}$$

and using (A.7), (A.8) and (A.17) to replace e_{w_0}, e_{q_0} and $\frac{\partial c_0}{\partial \widetilde{\omega}}$, we find

$$\begin{aligned} dc_0 &= \frac{MPC}{q_0}\left(q_0 n_0 dw_0 + q_0 dy_0 + URE dq_0 - q_0 NNP\frac{dP}{P}\right) + \frac{\partial c_0^h}{\partial q_0}dq_0 + \frac{\partial c_0^h}{\partial w_0}dw_0 \\ &= MPC\left(n_0 dw_0 + dy_0 + URE\frac{dq_0}{q_0} - NNP\frac{dP}{P}\right) + c_0\left(\frac{q_0}{c_0}\frac{\partial c_0^h}{\partial q_0}\frac{dq_0}{q_0} + \frac{w_0}{c_0}\frac{\partial c_0^h}{\partial w_0}\frac{dw_0}{w_0}\right) \end{aligned}$$

Finally, dropping time subscripts for ease of notation, using (A.5), and defining compensated elasticities by

$$\begin{aligned} \epsilon_{c,q}^h &\equiv \frac{q_0}{c_0}\frac{\partial c_0^h}{\partial q_0} \\ \epsilon_{c,w}^h &\equiv \frac{w_0}{c_0}\frac{\partial c_0^h}{\partial w_0} \end{aligned}$$

we obtain

$$dc = MPC\left(ndw + dy + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + c\left(\epsilon_{c,q}^h\frac{dR}{R} + \epsilon_{c,w}^h\frac{dw}{w}\right) \tag{A.19}$$

In a completely analogous way, we also find

$$dn = MPN\left(ndw + dy + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + n\left(\epsilon_{n,q}^h\frac{dR}{R} + \epsilon_{n,w}^h\frac{dw}{w}\right) \tag{A.20}$$

The rest of the proof calculates the compensated elasticities and relates them to MPC and MPN, which will yield our expressions for consumption and labor supply. To get my expression for welfare, totally differentiate the indirect utility function and use (A.12)–(A.14) and (A.10) to obtain

$$\begin{aligned} dU &= \frac{\partial V}{\partial q_0}dq_0 + \frac{\partial V}{\partial w_0}dw_0 + \frac{\partial V}{\partial \widetilde{\omega}}d\widetilde{\omega} \\ &= \frac{u'(c_0)}{q_0}\cdot\left(URE dq_0 + q_0 n_0 dw_0 + q_0 dy_0 - q_0 NNP\frac{dP}{P}\right) \end{aligned}$$

This yields my expression in (5),

$$dU = u'(c) \cdot \left(dy + ndw + URE\frac{dR}{R} - NNP\frac{dP}{P} \right)$$

Step 2: Marginal propensities. I now derive explicit expressions for marginal propensities to consume, that is, the Marshallian derivatives of the consumption and labor supply functions that are solutions to (A.11). Inverting the first-order conditions

$$u'(c_t) = \beta^{-t}\left(\frac{q_t}{q_0}\right)u'(c_0) \tag{A.21}$$

$$v'(n_t) = \beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)v'(n_0) \tag{A.22}$$

and inserting the resulting values for c_t and n_t into the budget constraint (redefining $W = q_0\widetilde{\omega}$ as present-value wealth for simplicity)

$$\sum_{t\geq 0}\frac{q_t}{q_0}(c_t - w_t n_t) = W$$

we obtain

$$c_0 + \sum_{t\geq 1}\frac{q_t}{q_0}(u')^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)u'(c_0)\right] - w_0\left(n_0 + \sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}(v')^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)v'(n_0)\right]\right) = W \tag{A.23}$$

Recall that $MPC = \frac{\partial c_0}{\partial W}$ and $MPN = \frac{\partial n_0}{\partial W}$. Differentiating (A.23) with respect to W, we obtain

$$MPC\left(1 + \sum_{t\geq 1}\frac{q_t}{q_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)}\right) - w_0 MPN\left(1 + \sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right) = 1 \tag{A.24}$$

moreover, the intratemporal first order condition

$$v'(n_0) = w_0 u'(c_0) \tag{A.25}$$

implies

$$\begin{aligned} v''(n_0)\,MPN &= w_0 u''(c_0)\,MPC \\ \frac{v''(n_0)}{v'(n_0)}MPN &= \frac{u''(c_0)}{u'(c_0)}MPC \end{aligned}$$

so, using the definition of the local elasticities of substitution,

$$-\sigma(c_t)\,c_t u''(c_t) = u'(c_t) \tag{A.26}$$

$$\psi(n_t)\,n_t v''(n_t) = v'(n_t) \tag{A.27}$$

we see that MPC and MPN are related through

$$MPN = -\frac{\psi(n_0)}{\sigma(c_0)}\frac{n_0}{c_0}MPC$$

Inserting into (A.24), this gives

$$MPC = \left(1 + \sum_{t\geq 1} \frac{q_t}{q_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)} + \frac{\psi(n_0)}{\sigma(c_0)}\frac{w_0 n_0}{c_0}\sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right)^{-1} \tag{A.28}$$

as well as

$$\begin{aligned} MPS &= 1 - MPC + w_0 MPN \\ &= MPC\left(\sum_{t\geq 1}\frac{q_t}{q_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)}\right. \\ &\qquad \left. + \frac{\psi(n_0)}{\sigma(c_0)}\frac{w_0 n_0}{c_0}\sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right) \end{aligned} \tag{A.29}$$

Expressions (A.28) and (A.29) can also be rewritten using the fact that (A.21)-(A.22) together with (A.26)-(A.27) yield

$$\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)} = \frac{\sigma(c_t)\,c_t}{\sigma(c_0)\,c_0} \qquad \beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)} = \frac{\psi(n_t)\,n_t}{\psi(n_0)\,n_0}$$

So, we also have

$$MPC = \left(1 + \sum_{t\geq 1}\frac{q_t}{q_0}\frac{\sigma(c_t)\,c_t}{\sigma(c_0)\,c_0} + \frac{\psi(n_0)}{\sigma(c_0)}\frac{w_0 n_0}{c_0}\left(1 + \sum_{t\geq 1}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{\psi(n_t)\,n_t}{\psi(n_0)\,n_0}\right)\right)^{-1}$$

Step 3: Hicksian elasticities. The solution to the expenditure minimization problem in (A.6) also involves the first-order conditions (A.21)-(A.22) , from which we obtain

$$u(c_t) = u\left((u')^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)u'(c_0)\right]\right) \qquad v(n_t) = v\left((v')^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)v'(n_0)\right]\right)$$

attaining utility U requires that the initial values c_0, n_0 satisfy

$$\begin{aligned} u(c_0) + \sum_{t\geq 1}\beta^t u\left((u')^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)u'(c_0)\right]\right) - v(n_0) \\ - \sum_{t\geq 1}\beta^t v\left((v')^{-1}\left[\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)v'(n_0)\right]\right) = U \end{aligned} \tag{A.30}$$

Differentiating with respect to q_0 along the indifference curve (A.30) results in

$$\begin{aligned} &\frac{\partial c_0}{\partial q_0}\left(u'(c_0) + \sum_{t\geq 1}\beta^t u'(c_t)\,\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)}\right) \\ &-\frac{\partial n_0}{\partial q_0}\left(v'(n_0) + \sum_{t\geq 1}\beta^t v'(n_t)\,\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right) \\ -\sum_{t\geq 1}\beta^t\frac{u'(c_t)}{u''(c_t)}&\left(\beta^{-t}\frac{q_t}{q_0^2}u'(c_0)\right) - \sum_{t\geq 1}\beta^t\frac{v'(n_t)}{v''(n_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}\left(\frac{w_t}{w_0}\right)v'(n_0)\right) = 0 \end{aligned}$$

dividing by $u'(c_0)$ and using (A.21), (A.25), (A.26) and (A.27) we find

$$\frac{\partial c_0}{\partial q_0}\left(1+\sum_t \frac{q_t}{q_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)}\right)-\frac{\partial n_0}{\partial q_0}w_0\left(1+\sum_{t\geq 1}\frac{q_t}{q_0}\frac{w_t}{w_0}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right)$$
$$=\frac{1}{u'(c_0)}\left(\sum_{t\geq 1}\beta^t\frac{u'(c_t)}{u''(c_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}u'(c_0)\right)+\sum_{t\geq 1}\beta^t\frac{v'(n_t)}{v''(n_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}\left(\frac{w_t}{w_0}\right)v'(n_0)\right)\right)$$

moreover, differentiating (A.25) we also find

$$\frac{\partial n_0}{\partial q_0}=-\frac{\psi(n_0)}{\sigma(c_0)}\frac{n_0}{c_0}\frac{\partial c_0}{\partial q_0}$$

Gathering results, we recognize, on the left-hand-side, the MPC expression in (A.28). We then use first-order conditions on the right hand side to obtain

$$\begin{aligned}\frac{\partial c_0}{\partial q_0}MPC^{-1} &= \frac{1}{u'(c_0)}\left\{\sum_{t\geq 1}\beta^t\frac{u'(c_t)}{u''(c_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}u'(c_0)\right)-\sum_{t\geq 1}\beta^t\frac{v'(n_t)}{v''(n_t)}\left(\beta^{-t}\frac{q_t}{q_0^2}\left(\frac{w_t}{w_0}\right)v'(n_0)\right)\right\}\\ &= \frac{1}{q_0}\left(\sum_{t\geq 1}\frac{u'(c_t)}{u''(c_t)}\frac{q_t}{q_0}-w_0\sum_{t\geq 1}\frac{v'(n_t)}{v''(n_t)}\frac{q_t}{q_0}\left(\frac{w_t}{w_0}\right)\right)\end{aligned}$$

Manipulating the right-hand side, we recognize the expression for (A.29) as

$$\begin{aligned}\frac{\partial c_0}{\partial q_0}MPC^{-1} &= -\frac{1}{q_0}\sigma(c_0)c_0\left\{\sum_{t\geq 1}\beta^{-t}\left(\frac{q_t}{q_0}\right)\frac{u''(c_0)}{u''(c_t)}\frac{q_t}{q_0}\right.\\ &\quad\left.+\frac{w_0n_0}{c_0}\frac{\psi(n_0)}{\sigma(c_0)}\sum_{t\geq 1}\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\frac{q_t}{q_0}\left(\frac{w_t}{w_0}\right)\right\}\\ &= -\frac{1}{q_0}\sigma(c_0)c_0\frac{MPS}{MPC}\end{aligned}$$

and therefore, we finally simply have

$$\left.\frac{\partial c_0}{\partial q_0}\right|_U = -\frac{c_0}{q_0}\sigma(c_0)MPS$$

which corresponds to a Hicksian elasticity of

$$\epsilon^h_{c_0,q_0}=-\sigma(c_0)MPS \tag{A.31}$$

A similar procedure can be used to differentiate with respect to w_0: from (A.25) we obtain

$$\frac{\partial n_0}{\partial w_0}=-\frac{\psi(n_0)}{\sigma(c_0)}\frac{n_0}{c_0}\frac{\partial c_0}{\partial q_s}+\psi(n_0)\frac{n_0}{w_0}$$

and differentiating along (A.28) we therefore obtain

$$\frac{\partial c_0}{\partial w_0}u'(c_0)MPC^{-1}+\psi(n_0)\frac{n_0}{w_0}\left(v'(n_0)+\sum_{t\geq 1}\beta^t v'(n_t)\beta^{-t}\left(\frac{q_t}{q_0}\right)\left(\frac{w_t}{w_0}\right)\frac{v''(n_0)}{v''(n_t)}\right)$$
$$=\sum_{t\geq 1}\beta^t\frac{v'(n_t)}{v''(n_t)}\beta^{-t}\frac{q_t}{q_0}\left(\frac{w_t}{w_0^2}\right)v'(n_0)$$

We conclude by noticing that $v'(n_0) = \psi(n_0) n_0 v''(n_0)$, so

$$\left.\frac{\partial c_0}{\partial w_0}\right\rfloor_U = MPC\psi(n_0) n_0$$

and

$$\epsilon^h_{c_0,w_0} = MPC\left(\psi(n_0)\frac{w_0 n_0}{c_0}\right) \quad \text{(A.32)}$$

Finally, elasticities for n_0 result from a final differentiation of (A.25):

$$\begin{aligned}
\epsilon^h_{n_0,q_0} &= -\frac{\psi(n_0)}{\sigma(c_0)}\epsilon^h_{c_0,q_0} && \text{(A.33)}\\
\epsilon^h_{n_0,w_0} &= \psi(n_0)\left(1 - \frac{1}{\sigma(c_0)}\epsilon^h_{c_0,w_0}\right)\\
&= \psi(n_0)\left(1 - \frac{\psi(n_0)}{\sigma(c_0)}\frac{w_0 n_0}{c_0}MPC\right)\\
&= \psi(n_0)(1 + w_0 MPN) && \text{(A.34)}
\end{aligned}$$

Step 4: Putting all expressions together. For consumption, equations (A.31)–(A.32) can be inserted into (A.19) to yield

$$dc = MPC\left(ndw + dy + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + c\left(-\sigma MPS\frac{dR}{R} + \psi MPC\frac{wn}{c}\frac{dw}{w}\right)$$

The first term is the wealth effect, and the last two terms the substitution effects with respect to interest rates and wages. We then simplify the expression to

$$dc = MPC\left(dy + n(1+\psi)dw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \sigma c MPS\frac{dR}{R} \quad \text{(A.35)}$$

which is our equation (3).

Similarly, equations (A.33)–(A.34) can be inserted into (A.20) to yield

$$dn = MPN\left(ndw + dy + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + n\left(-\psi MPS\frac{dR}{R} + \psi(1 + wMPN)\frac{dw}{w}\right)$$

and we again naturally separate the latter piece to obtain

$$dn = MPN\left(dy + n(1+\psi)dw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \psi n MPS\frac{dR}{R} + \psi n\frac{dw}{w} \quad \text{(A.36)}$$

which is equation (4).

A.3 Extension of Theorem 1 to general preferences and persistent changes

Theorem 1 in the main text is a special case of a general decomposition that holds for arbitrary nonsatiable preferences U over $\{c_t\}$ and $\{n_t\}$ and for any change in the price level $\{P_0, P_1 \ldots\}$, the real term structure $\{q_0 = 1, q_1, q_2 \ldots\}$, the agent's unearned income sequence $\{y_0, y_1 \ldots\}$ and the stream of real wages $\{w_0, w_1 \ldots\}$, with the nominal term structure adjusting instantaneously to make the Fisher equation hold at the post-shock sequences of interest rates and prices. The utility maximization problem is then

$$\begin{aligned} \max \quad & U(\{c_t, n_t\}) \\ \text{s.t.} \quad & P_t c_t = P_t y_t + W_t n_t + ({}_{t-1}B_t) + \sum_{s \geq 1} ({}_tQ_{t+s})({}_{t-1}B_{t+s} - {}_tB_{t+s}) \\ & \qquad + P_t({}_{t-1}b_t) + \sum_{s \geq 1} ({}_tq_{t+s}) P_{t+s} ({}_{t-1}b_{t+s} - {}_tb_{t+s}) \end{aligned}$$

and the first order date-0 responses of consumption, labor supply and welfare to the considered change are, in this case, given by

$$\begin{aligned} dc_0 &= MPC d\Omega + c_0 \left(\sum_{t \geq 0} \epsilon^h_{c_0, q_t} \frac{dq_t}{q_t} + \sum_{t \geq 0} \epsilon^h_{c_0, w_t} \frac{dw_t}{w_t} \right) \\ dn_0 &= MPN d\Omega + n_0 \left(\sum_{t \geq 0} \epsilon^h_{n_0, q_t} \frac{dq_t}{q_t} + \sum_{t \geq 0} \epsilon^h_{n_0, w_t} \frac{dw_t}{w_t} \right) \\ dU &= U_{c_0} d\Omega \end{aligned}$$

where $\epsilon^h_{x_0, y_t} = \frac{\partial x^h_0}{\partial y_t} \frac{y_t}{x_0}$ for $x \in \{c, n\}$ and $y \in \{q, w\}$ are Hicksian elasticities and $d\Omega = dW - \sum_{t \geq 0} c_t dq_t$, the net-of-consumption wealth change, is given by

$$\begin{aligned} d\Omega \quad = \quad & \underbrace{\sum_{t \geq 0} (q_t y_t) \frac{dy_t}{y_t}}_{\text{Real unearned income change}} + \underbrace{\sum_{t \geq 0} (q_t w_t n_t) \frac{dw_t}{w_t}}_{\text{Real earned income change}} \\ & + \underbrace{\sum_{t \geq 0} q_t \left(y_t + w_t n_t + \left(\frac{{}_{-1}B_t}{P_t} \right) + ({}_{-1}b_t) - c_t \right) \frac{dq_t}{q_t}}_{\text{Revaluation of net savings flows}} \\ & - \underbrace{\sum_{t \geq 0} Q_t \left(\frac{{}_{-1}B_t}{P_0} \right) \frac{dP_t}{P_t}}_{\text{Revaluation of net nominal position}} \end{aligned} \tag{A.37}$$

The proof is a generalization of that in section A.2. I omit it here in the interest of space.

Values of all elasticities with separable preferences in a steady-state with no growth. Following once more the steps of section A.2, it is possible to derive the value of Hicksian elasticities for a change at any horizon. Here I just report the values of these elasticities in the case of an infinite horizon model where $\frac{q_s}{q_0} = \beta^s$ and $w_s = w^*$, $\forall s$. These prices correspond to those prevailing in a steady-state with no growth of any such model, and the resulting elasticities are relevant, for example, to determine the impulse responses in many RBC and DSGE models. The first order conditions imply that consumption and labor supply are constant. Let us call the solutions c^* and n^*, respectively. Writing $\vartheta \equiv \frac{w^* n^*}{c^*}$ for the share of earned income in consumption and $\kappa \equiv \frac{\frac{\psi}{\sigma}\vartheta}{1+\frac{\psi}{\sigma}\vartheta} \in (0,1)$, obtain values of elasticities summarized in table A.1.

Table A.1: Steady-state moments, separable preferences

ϵ^h	q_0	$q_s, s \geq 1$	w_0	$w_s, s \geq 1$	Marg. propensity	
c_0	$-\sigma\beta$	$\sigma(1-\beta)\beta^s$	$\sigma\kappa(1-\beta)$	$\sigma\kappa(1-\beta)\beta^s$	MPC	$(1-\kappa)(1-\beta)$
n_0	$\psi\beta$	$-\psi(1-\beta)\beta^s$	$\psi(1-\kappa(1-\beta))$	$-\psi\kappa(1-\beta)\beta^s$	MPN	$-\frac{1}{w^*}\kappa(1-\beta)$
					MPS	$(1-\beta)$

A.4 Proof of Corollary 1

Rewrite equations (A.35) and (A.36) as

$$\begin{aligned} dc &= MPC\left(dY + \psi dw - wdn + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \sigma cMPS\frac{dR}{R} \\ wdn - \psi ndw &= wMPN\left(dY + \psi dw - wdn + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + \psi wnMPS\frac{dR}{R} \end{aligned}$$

Hence

$$wdn - \psi ndw = \frac{1}{1+wMPN}\left\{wMPN\left(dY + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + \psi wnMPS\frac{dR}{R}\right\}$$

which, inserted into the expression for dc yields

$$\begin{aligned} dc = MPC\left(1 - \frac{wMPN}{1+wMPN}\right)\left(dY + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) \\ - \sigma cMPS\left(1 + MPC\frac{\psi wn}{\sigma c}\frac{1}{1+wMPN}\right)\frac{dR}{R} \end{aligned}$$

But $MPC\frac{\psi n}{\sigma c} = -MPN$ so this is

$$dc = \left(\frac{MPC}{1+wMPN}\right)\left(dY + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \sigma c\frac{MPS}{1+wMPN}\frac{dR}{R}$$

and noting that

$$1 + wMPN = MPC + MPS$$

we can finally rewrite this in terms of $M\hat{P}C = \frac{MPC}{MPC+MPS}$ as

$$dc = M\hat{P}C\left(dY + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \sigma c\left(1 - M\hat{P}C\right)\frac{dR}{R}$$

as claimed.

A.5 Adding durable goods

This section shows the consequences of adding durable goods to the model.

I consider a standard durable goods problem. For simplicity, I ignore labor supply and nominal assets, neither of which interacts with the conclusions below. A consumer maximizes a separable intertemporal utility function

$$\begin{aligned}
\max \quad & \sum \beta^t \left\{u\left(C_t\right) + w\left(D_t\right)\right\} \\
\text{s.t.} \quad & C_t + p_t I_t = Y_t + \left({}_{t-1}b_t\right) + \sum_{s\geq 1}\left({}_t q_{t+s}\right)\left({}_{t-1}b_{t+s} - {}_t b_{t+s}\right) \\
& D_t = I_t + D_{t-1}\left(1-\delta\right) \\
& D_{-1}, \left\{{}_{-1}b_t\right\} \qquad \text{given}
\end{aligned}$$

where C_t is now *nondurable* consumption, D_t is the consumer's stock of durables, and p_t is the relative price of durable goods in period t.

I am interested in the response of the demand for nondurable goods C_t and durables goods I_t, as well as that of total expenditures

$$X_t \equiv C_t + p_t I_t \tag{A.38}$$

to a change in the time-0 nondurable real interest rate R_0 and (potentially) a simultaneous change in the price of durables p_0. As I argue below, the notion of *aggregate demand* makes most sense when the relative price of durables does not change with R_0, but I start by covering the general case in which p_0 can change.

The intertemporal budget constraint reads

$$\sum_{t\geq 0} q_t\left(C_t+p_tI_t\right)=\sum_{t\geq 0} q_tY_t+\sum_{t\geq 0} q_t\left({}_{-1}b_t\right)$$

Defining $R_t \equiv \frac{q_t}{q_{t+1}}$, the first-order conditions of this problem are, for all $t \geq 0$

$$u'\left(C_t\right) = \beta R_t u'\left(C_{t+1}\right) \tag{A.39}$$

$$w'\left(D_t\right) = u'\left(C_t\right)\left[p_t-\frac{\left(1-\delta\right)p_{t+1}}{R_t}\right] \tag{A.40}$$

Equation (A.39) is the standard Euler equation for nondurable consumption. Equation (A.40) shows that the consumer equates the marginal rate of substitution between the stock of durables and consumption to the user cost of durables, $p_t-\frac{(1-\delta)p_{t+1}}{R_t}$. A fall in the nondurable real interest rate at date 0, R_0, increases the desired level of nondurable consumption and of the stock of nondurables (an intertemporal substitution effect). Holding p_1 constant, it also reduces the user cost of durables, increasing the desired stock of durables relative to nondurable consumption. A fall in p_0 has the same effect of reducing the durable user cost, but it does not affect intertemporal substitution in consumption.

Suppose that the path for interest rates $\{R_t\}$, relative prices $\{p_t\}$ and income $\{Y_t\}$ delivers the solution $\{C_t, D_t\}$. Consider the solution under the alternative paths $\left\{\overline{R_0}, R_1, R_2\ldots\right\}$, $\{\overline{p_0}, p_1, p_2\ldots\}$, and $\left\{\overline{Y_0}, Y_1, Y_2\ldots\right\}$. Let $dR=\overline{R_0}-R_0$, $dp=\overline{p_0}-p_0$ and $dY=\overline{Y_0}-Y_0$. I am interested in the response of the paths of nondurable and durable expenditures to these changes. To obtain this, I find the paths for consumption $\{C_t\}$ and durables $\{D_t\}$, and then find the implied path for durable expenditures $\{p_tI_t\}$.

Marshallian demand. In order to determine the Marshallian demands, I could follow the same proof as that of section A.2, but here I follow an alternative and somewhat more intuitive procedure. The procedure is in two steps. First, I determine a variation that respects all the first-order conditions (A.39)–(A.40) at the new prices. This gives dC^* and dD^*, which result in a budgetary cost $d\Omega^*$ at the old prices. Second, I determine the change in net wealth $d\Omega$ that results from the change in prices. The Marshaling demands are then

$$dC = dC^*+MPC\left(d\Omega-d\Omega^*\right) \tag{A.41}$$

$$dD = dD^*+MPD\left(d\Omega-d\Omega^*\right) \tag{A.42}$$

where $MPD=\frac{\partial D}{\partial Y}$ is the increase in the stock of date-0 durables that results from a date-0 increase in income. Note that MPC and MPD are related: differentiating

(A.40), we find

$$w''(D_0)\,MPD = u''(C_0)\,MPC\left[p_0 - \frac{(1-\delta)\,p_1}{R_0}\right]$$

so

$$MPD = \frac{\sigma_D}{\sigma_C}\frac{D_0}{C_0}MPC$$

where $\sigma_C \equiv -\frac{u'(C_0)}{u''(C_0)C_0}$ and $\sigma_D \equiv -\frac{w'(D_0)}{w''(D_0)D_0}$ are the elasticities of intertemporal substitution in consumption and in the stock of durables. Since $D_0 = I_0 + D_{-1}(1-\delta)$ and the initial stock D_{-1} is fixed, the total constant-p marginal propensity to spend at date 0 is

$$\begin{aligned} MPX \equiv \frac{\partial (C+pI)}{\partial Y} = \frac{\partial C}{\partial Y} + p\frac{\partial D}{\partial Y} &= MPC + pMPD \\ &= MPC\left(1 + \frac{\sigma_D}{\sigma_C}\frac{pD}{C}\right) \end{aligned}$$

Step 1: variation respecting FOCs. The simplest variation that respects all FOCs holds the paths $\{C_t\}$ and $\{D_t\}$ fixed for all $t \geq 1$ and adjusts C_0 and D_0 by dC (respectively dD) such that (A.39) and (A.40) are satisfied at $t = 0$. Differentiating these equations, I obtain

$$\begin{aligned} -\frac{1}{\sigma_C}\frac{dC}{C} &= \frac{dR}{R} \\ -\frac{1}{\sigma_D}\frac{dD}{D} &= -\frac{1}{\sigma_C}\frac{dC}{C} + \frac{p_1\frac{1-\delta}{R}}{p_0 - p_1\frac{1-\delta}{R}}\frac{dR}{R} + \frac{p_0}{p_0 - p_1\frac{1-\delta}{R}}\frac{dp}{p} \end{aligned}$$

Hence we find

$$dC^* = -\sigma_C C\frac{dR}{R} \tag{A.43}$$

and

$$dD^* = -\sigma_D D\left[\frac{p_0}{p_0 - p_1\frac{1-\delta}{R}}\right]\left(\frac{dR}{R} + \frac{dp}{p}\right) \tag{A.44}$$

These responses are very intuitive: one way to respond to a fall in real interest rates is to raise nondurable consumption and the stock of durables. The relevant elasticity for durables is higher than σ_D because of the additional substitution effect coming from the change in the user cost. A lower current relative price of durables has a symmetric effect on the demand for durables as that of a lower real interest rate (in other words, it is the real interest rate in terms of durables that matters for durables demand).

We are now ready to determine the net cost of this variation. Since

$$\begin{aligned} D_0 &= (1-\delta) D_{-1} + I_0 \\ D_1 &= (1-\delta) D_0 + I_1 \end{aligned}$$

the sequence of investment that achieves this variation consists naturally in an increase of dD^* followed by a subsequent decrease:

$$\begin{aligned} dI_0^* &= dD^* \\ dI_1^* &= -(1-\delta) dD^* \end{aligned}$$

Hence the total budgetary cost of this 'star' variation at the old prices p and R has the simple form

$$\begin{aligned} d\Omega^* &= dC^* + p_0 dI_0^* + p_1 \frac{dI_1^*}{R} \\ &= dC^* + \left(p_0 - p_1 \frac{1-\delta}{R}\right) dD^* \\ &= -(\sigma_C C + p_0 \sigma_D D) \frac{dR}{R} - \sigma_D p_0 D \frac{dp}{p} \end{aligned}$$

Step 2: change in net wealth. Let Ω be defined as

$$\Omega \equiv \sum_{t \geq 0} q_t \left\{ Y_t + ({}_{-1}b_t) - C_t - p_t I_t \right\}.$$

At the initial prices, the intertemporal budget constraint implies $\Omega = 0$. The exogenous variation dR, dp and dY yields

$$\begin{aligned} d\Omega &= dY - I dp + \sum_{t \geq 0} dq_t \left\{ Y_t + ({}_{-1}b_t) - C_t - p_t I_t \right\} \\ &= dY - I dp - \sum_{t \geq 1} q_t \left\{ Y_t + ({}_{-1}b_t) - C_t - p_t I_t \right\} \frac{dR}{R} \\ &= dY - p I_0 \frac{dp}{p} + \left(\underbrace{Y_0 + ({}_{-1}b_0) - C_0 - p_0 I_0}_{URE} \right) \frac{dR}{R} \end{aligned} \tag{A.45}$$

The intuition is as follows. Suppose that the nondurable real interest rate falls at date 0. As before, this benefits consumers that have a negative URE, that is, maturing liabilities $C_0 + p_0 I_0$ in excess maturing assets $Y_0 + ({}_{-1}b_0)$. Note that, for this effect, total expenditures *including expenditures on durables* are counted as part of URE. In that sense, URE measures the true balance-sheet exposure to a change in the real interest rate. In particular, ceteris paribus, when investment is higher today the consumer benefits more from a fall in real interest rates.

Suppose however that, in parallel, the relative price of durables rises. In the general equilibrium model of Barsky et al. (2007), for example, this happens in response to an accommodative monetary policy shock when durable goods prices are more flexible than nondurable goods prices. In that case, equation (A.45) shows that there is an additional capital loss on wealth due to the rise in the durable relative price. While conceptually distinct, these two effects could be consolidated into a single one, if we restrict ourselves to variations that feature a constant elasticity of the durable-good price to the nondurable real interest rate

$$\epsilon_{pR} \equiv -\frac{\partial p}{p}\frac{R}{\partial R} \tag{A.46}$$

The benchmark case where p is constant corresponds to $\epsilon_{pR} = 0$, the case where the durable real interest rate is constant to $\epsilon_{pR} = 1$. Then,

$$d\Omega = dY + \left(\underbrace{Y_0 + ({}_{-1}b_0) - C_0 - p_0 I_0 \left(1 - \epsilon_{pR}\right)}_{URE^{\epsilon}}\right)\frac{dR}{R} \tag{A.47}$$

In other words, once we net out the capital revaluation effect, an alternative measure of URE becomes URE^{ϵ}, which subtracts a fraction $(1 - \epsilon_{pR})$ of durable expenditures.

Step 3: demand for durables and nondurables. Combining (A.41)–(A.42) with (A.43), (A.44) and (A.45), I obtain the Marshallian demands (recall that $dI = dD$ at time 0)

$$\begin{aligned}
dC &= MPC\left(dY + URE\frac{dR}{R} + \left(\sigma_C C + p\sigma_D D\right)\frac{dR}{R} + \left(p\sigma_D D - pI_0\right)\frac{dp}{p}\right) - \sigma_C C\frac{dR}{R} \\
dD &= MPD\left(dY + URE\frac{dR}{R} + \left(\sigma_C C + p\sigma_D D\right)\frac{dR}{R} + \left(p\sigma_D D - pI_0\right)\frac{dp}{p}\right) \\
&\quad -\sigma_D D\left[\frac{p_0}{p_0 - p_1\frac{1-\delta}{R}}\right]\left(\frac{dR}{R} + \frac{dp}{p}\right)
\end{aligned}$$

This separates out the separate effects from changing R and p. Given the elasticity ϵ_{pR} in (A.46), we can also rewrite this as

$$\begin{aligned}
dC &= MPC\left(dY + URE^{\epsilon}\frac{dR}{R}\right) - \sigma_C C\left(1 - MPC\right)\frac{dR}{R} \\
&\quad +\sigma_D \cdot pD \cdot MPC \cdot \left(1 - \epsilon_{pR}\right) \cdot \frac{dR}{R}
\end{aligned} \tag{A.48}$$

$$\begin{aligned}
dD &= MPD\left(dY + URE^{\epsilon}\frac{dR}{R}\right) + \sigma_C \cdot MPD \cdot C \cdot \frac{dR}{R} \\
&\quad -\sigma_D \cdot pD \cdot \left(1 - \epsilon_{pR}\right) \cdot \left(1 - MPD\right) \cdot \left[\frac{1}{p_0 - p_1\frac{1-\delta}{R}}\right]\frac{dR}{R}
\end{aligned} \tag{A.49}$$

Where URE^{ϵ} is defined in (A.47).

Special case with constant durable real interest rate ($\epsilon_{pR} = 1$). When $\epsilon_{pR} = 1$, equations (A.48)–(A.49) simplify to

$$\begin{aligned} dC &= MPC\left(dY + URE^{1}\frac{dR}{R}\right) - \sigma_C C\left(1 - MPC\right)\frac{dR}{R} \\ dD &= MPD\left(dY + URE^{1}\frac{dR}{R}\right) + \sigma_C \cdot MPD \cdot C \cdot \frac{dR}{R} \end{aligned}$$

which are simple extensions of expressions in the main text, with URE^{1} (which does not subtract durable expenditures) replacing URE. Note that to the extent that $URE^{1} \geq 0$, the expression for dD implies a contraction in durable goods from an increase in real interest rates, as in Barsky et al. (2007). This is counterfactual, suggesting that $\epsilon_{pR} = 1$ may be too high an elasticity in practice.

Special case with constant relative price ($\epsilon_{pR} = 0$). While the cases where $\epsilon_{pR} \neq 0$ are interesting in principle, they prevent a straightforward definition of *aggregate demand* $X = C + pI$: if the relative price of two goods can change, then the relative demands for these two goods (as well as their relative supplies) will matter for general equilibrium. Therefore, the case where $\epsilon_{pR} = 0$ is the most relevant for my purposes. Assume then that $p_0 = p_1 = p$. In this case, we can combine (A.48) and (A.49) to obtain an expression for the change in aggregate demand $dX = dC + pdD$ as a function of the marginal propensity to spend $MPX = MPC + pMPD$ and other variables

$$dX = MPX\left(dY + URE\frac{dR}{R} + \sigma_C C + \sigma_D pD\right) - \left(\sigma_C C + \frac{\sigma_D pD}{1 - \frac{1-\delta}{R}}\right)\frac{dR}{R}$$

This can further be simplified to yield an expression with the same form as the expression in the main text,

$$dX = MPX\left(dY + URE\frac{dR}{R}\right) - \sigma_X\left(1 - MPX\right)X\frac{dR}{R} \tag{A.50}$$

where σ_X is defined as

$$\sigma_X \equiv \frac{C}{X}\cdot\sigma_C + \left(1 - \frac{C}{X}\right)\cdot\sigma_D\cdot\frac{pD}{pI}\cdot\frac{\frac{1}{1-\frac{1-\delta}{R}} - MPX}{1 - MPX} \tag{A.51}$$

In other words, σ_X is a weighted average of σ_C and the relevant elasticity of substitution in durable expenditures: the product of σ_D by the stock-flow ratio $\frac{pD}{pI}$, multiplied by a term that increases in the elasticity of the user cost to the real interest rate.

Quantitatively, the second term is likely to be much larger than the first. If initially durable expenditures cover replacement costs $I = D\delta$, then the stock-flow ratio is $\frac{1}{\delta}$. Hence, with $\delta = 5\%$ and $R = 1.05$ at annual rates, the second term in (A.51) is at least as large as $\frac{1}{20} \times \frac{1}{10} \times \sigma_D = 200\sigma_D$. This makes aggregate demand very sensitive to given changes in the real interest rate because of the large substitution effect that results from the presence of long-lived durables, a point made by Barsky et al. (2007).

A.6 Proof of Theorem 2

After dividing through by P_t, defining the real bond position as $\lambda_t \equiv \frac{\Lambda_t}{P_{t-1}}$ and writing $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ for the inflation rate between $t-1$ and t, the budget constraint (9) becomes

$$c_t + Q_t\left(\lambda_{t+1} - \delta\frac{\lambda_t}{\Pi_t}\right) + (\theta_{t+1} - \theta_t)\cdot\mathbf{S}_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t\cdot\mathbf{d}_t$$

In this notation, the consumer's date-t net nominal position is

$$NNP_t = (1 + Q_t\delta)\frac{\lambda_t}{\Pi_t}$$

while his unhedged interest rate exposure is:

$$URE_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t\cdot\mathbf{d}_t - c_t = Q_t\left(\lambda_{t+1} - \delta\frac{\lambda_t}{\Pi_t}\right) + (\theta_{t+1} - \theta_t)\cdot\mathbf{S}_t$$

His optimization problem can be represented using the recursive formulation

$$\max_{c,n,\lambda',\theta'} u(c) - v(n) + \underbrace{\beta\mathbb{E}\left[V\left(\lambda',\theta';y',w',Q',\Pi',\mathbf{d}',\mathbf{S}'\right)\right]}_{\equiv W(\lambda',\theta')}$$

$$\text{s.t.} \quad c + Q\left(\lambda' - \delta\frac{\lambda}{\Pi}\right) + (\theta' - \theta)\,\mathbf{S} = y + wn + \frac{\lambda}{\Pi} + \theta\mathbf{d} \qquad \text{(A.52)}$$

$$Q\lambda' + \theta'\mathbf{S} \geq \frac{\overline{D}}{R}$$

The function V corresponds to the value from optimizing given a starting real level of bonds λ' and shares θ', and includes the possibility of hitting future borrowing constraints.

I consider the predicted effects on c and n resulting from a simultaneous unexpected change in unearned income dy, the real wage dw, the price level $\frac{dP}{P} = \frac{d\Pi}{\Pi}$ and the real interest rate dR, which result in a change in asset prices $\frac{dQ}{Q} = \frac{dS_j}{S_j} = -\frac{dR}{R}$ for $j = 1\ldots N$. By leaving the future unaffected, this purely transitory change does not alter the value from future optimization starting at (λ',θ')— that is, the function W is unchanged. I claim that, provided the consumption and labor supply functions are differentiable, their first order differentials are

$$dc = MPC\left(dy + n\left(1+\psi\right)dw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) - \sigma c MPS\frac{dR}{R} \tag{A.53}$$

$$dn = MPN\left(dy + n\left(1+\psi\right)dw + URE\frac{dR}{R} - NNP\frac{dP}{P}\right) + \psi n MPS\frac{dR}{R} + \psi n\frac{dw}{w} \tag{A.54}$$

where $\sigma \equiv -\frac{u'(c)}{cu''(c)}$ and $\psi = \frac{v'(n)}{nv''(n)}$ are the local elasticities of intertemporal substitution and labor supply, respectively, $MPC = \frac{\partial c}{\partial y}$, $MPN = \frac{\partial n}{\partial y}$ and $MPS = 1 - MPC + wMPN$.

In order to prove (A.53) and (A.54), there are two cases to consider. In the first case, the consumer is at a binding borrowing limit or lives hand-to-mouth. The problem is then a static choice between c and n. In the second case, the consumer is at an interior optimum. The result then follows from application of the implicit function theorem to the set of $N+2$ first-order conditions which, together with the budget constraint, characterize the solution to the problem in (A.52). Here, to simplify the notation and the proof, I first prove the statement in the case where all variables are changing but $N = 0$, and then consider the case with stocks ($N > 0$) but without bonds and assuming only R is changing.

Case 1. Binding borrowing limit and hand-to-mouth agents.

Proof. The consumption of an agent at the borrowing limit is given by

$$c = wn + Z \tag{A.55}$$

where

$$Z = z + \left(1+Q\delta\right)\frac{\lambda}{\Pi} + \theta\cdot\left(\mathbf{d}+\mathbf{S}\right) + \frac{\overline{D}}{R}$$

Similarly, the consumption of an agent that lives hand to mouth is

$$c = wn + z$$

Given that $d\mathbf{S} = -\frac{\mathbf{S}}{R}dR$, $dQ = -\frac{Q}{R}dR$ and $d\left(\frac{1}{\Pi}\right) = -\frac{1}{\Pi^2}d\Pi = -\frac{1}{\Pi}\frac{dP}{P}$, we have, if the agent is at the borrowing limit

$$dZ = dz - \underbrace{\left(1+Q\delta\right)\frac{\lambda}{\Pi}}_{\text{NNP}}\frac{dP}{P} + \underbrace{\left(Q\delta\frac{\lambda}{\Pi} + \theta\cdot\mathbf{S} + \frac{\overline{D}}{R}\right)}_{-\text{URE}}\left(-\frac{dR}{R}\right) \tag{A.56}$$

and, if the agent lives hand to mouth,

$$dZ = dz$$

but since that agent also has

$$NNP = URE = 0$$

equation (A.56) still applies. In both cases, the consumer is making a static choice between c and n given the budget constraint (A.55), and hence has $MPS = 0$. We can then apply the results of section A.2 to find

$$\begin{aligned} dc &= MPC\left(dZ + w\left(1+\psi\right)\right) \\ dn &= MPN\left(dZ + w\left(1+\psi\right)\right) + \psi n dw \end{aligned}$$

which yields the desired result. □

Case 2a). $N = 0$**, all variables changing.** I first prove the following lemma.

Lemma A.1. *Let* $c\left(z,w,q,b\right)$ *and* $n\left(z,w,q,b\right)$ *be the solution to the following separable consumer choice problem under concave preferences over current consumption* $u\left(c\right)$ *and assets* $V\left(a\right)$*, and convex preferences over hours worked* $v\left(n\right)$*:*

$$\begin{aligned} \max \quad & u\left(c\right) - v\left(n\right) + V\left(a\right) \\ \textit{s.t.} \quad & c + q\left(a-b\right) = wn + z \end{aligned}$$

Assume $c\left(\right)$ *and* $n\left(\right)$ *are differentiable. Then the first order differentials are*

$$\begin{aligned} dc &= MPC\left(dz + n\left(1+\psi\right)dw - \left(a-b\right)dq + qdb\right) - \sigma c MPS\frac{dq}{q} \\ dn &= MPN\left(dz + n\left(1+\psi\right)dw - \left(a-b\right)dq + qdb\right) + \psi n MPS\frac{dq}{q} + \psi n\frac{dw}{w} \end{aligned}$$

where $MPC = \frac{\partial c}{\partial z}$*,* $MPN = \frac{\partial n}{\partial z}$ *and* $MPS = 1 - MPC + wMPN = 1 - MPC\left(1 + \frac{wn}{c}\frac{\psi}{\sigma}\right)$*.*

Proof. The following first-order conditions are necessary and sufficient for optimality:

$$u'\left(c\right) = \frac{1}{w}v'\left(n\right) = \frac{1}{q}V'\left(a\right) \tag{A.57}$$

I first obtain the expression for MPC by considering an increase in income dz alone. Consider how that increase is divided between current consumption, leisure and assets. (A.57) implies

$$u''\left(c\right)dc = \frac{1}{w}v''\left(n\right)dn = \frac{1}{q}V''\left(a\right)da \tag{A.58}$$

where the changes dc, dn and da are related to dz through the budget constraint

$$dc + qda = wdn + dz \tag{A.59}$$

Define $MPC = \frac{\partial c}{\partial z}$, $MPN = \frac{\partial n}{\partial z}$ and $MPS = q\frac{\partial a}{\partial z}$. Then (A.58) implies

$$\begin{aligned}
\frac{MPN}{MPC} &= w\frac{u''(c)}{v''(n)} = \frac{u''(c)}{u'(c)}\frac{v'(n)}{v''(n)} = -\frac{n}{c}\frac{\psi}{\sigma} \\
\frac{MPS}{MPC} &= \frac{q^2 u''(c)}{V''(a)} = \frac{q}{c}\frac{V'(a)}{\sigma V''(a)}
\end{aligned}$$

where $\sigma \equiv -\frac{u'(c)}{cu''(c)}$ and $\psi \equiv \frac{v'(n)}{nv''(n)}$. Hence the total marginal propensity to spend is

$$1 - MPS = \frac{\partial c}{\partial z} - w\frac{\partial n}{\partial z} = MPC\left(1 + \frac{wn}{c}\frac{\psi(n)}{\sigma(c)}\right) = 1 - \frac{q^2 u''(c)}{V''(a)} MPC \tag{A.60}$$

and the marginal propensity to consume is

$$MPC = \frac{1}{1 + q^2\frac{u''(c)}{V''(a)} - w^2\frac{u''(c)}{v''(n)}} = \frac{V''(a)\, v''(n)}{V''(a)\, v''(n) + q^2 u''(c)\, v''(n) - w^2 u''(c)\, V''(a)}$$

Consider now the overall effect on c, n and a of a change in q, w, z and b. Applying the implicit function theorem to the system of equations

$$\begin{cases} v'(n) - wu'(c) = 0 \\ V'(a) - qu'(c) = 0 \\ c + q(a-b) - wn - z = 0 \end{cases}$$

results in the following expression for partial derivatives:

$$\begin{aligned}
&\begin{bmatrix} \frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial w} & \frac{\partial c}{\partial b} \\ \frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b} \\ \frac{\partial a}{\partial q} & \frac{\partial a}{\partial z} & \frac{\partial a}{\partial w} & \frac{\partial a}{\partial b} \end{bmatrix} \\
&= -\underbrace{\begin{bmatrix} -wu''(c) & v''(n) & 0 \\ -qu''(c) & 0 & V''(a) \\ 1 & -w & q \end{bmatrix}^{-1}}_{\equiv A} \begin{bmatrix} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{bmatrix}
\end{aligned} \tag{A.61}$$

now

$$\det(A) = v''(n)\, V''(a) - w^2 u''(c)\, V''(a) + q^2 u''(c)\, v''(n) = \frac{V''(a)\, v''(n)}{MPC}$$

and so

$$A^{-1} = \frac{MPC}{V''(a)\, v''(n)} \begin{bmatrix} wV''(a) & -v''(n)\, q & v''(n)\, V''(a) \\ q^2 u''(c) + V''(a) & -wqu''(c) & wu''(c)\, V''(a) \\ qwu''(c) & w^2 u''(c) - v''(n) & qu''(c)\, v''(n) \end{bmatrix}$$

therefore, the first row of (A.61)

$$\left[\begin{array}{cccc} \frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial w} & \frac{\partial c}{\partial b} \end{array}\right] = MPC \left[\begin{array}{ccc} -\frac{w}{v''(n)} & \frac{q}{V''(a)} & -1 \end{array}\right] \left[\begin{array}{cccc} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{array}\right] \tag{A.62}$$

Using (A.60) we find

$$-q\frac{u'(c)}{V''(a)}MPC = \frac{\sigma c}{q}q^2\frac{u''(c)}{V''(a)}MPC = \frac{\sigma c}{q}MPS$$

so that the first column of the matrix equation (A.62) reads

$$\frac{\partial c}{\partial q} = \frac{\sigma c}{q}MPS - (a-b)\,MPC$$

The second and fourth column of (A.62) yield directly

$$\begin{aligned} \frac{\partial c}{\partial z} &= MPC \\ \frac{\partial c}{\partial b} &= qMPC \end{aligned}$$

Finally, using (A.57) we have

$$w\frac{u'(c)}{v''(n)} = \frac{v'(n)}{v''(n)} = \psi n$$

so that the third column of (A.62) reads

$$\begin{aligned} \frac{\partial c}{\partial w} &= MPC\psi n + MPCn \\ &= MPC\,(1+\psi)\,n \end{aligned}$$

The first-order total differential dc is then

$$\begin{aligned} dc &= \frac{\partial c}{\partial z}dz + \frac{\partial c}{\partial b}db + \frac{\partial c}{\partial q}dq + \frac{\partial c}{\partial w}dw \\ &= MPC\,(dz + qdb - (a-b)\,dq + (1+\psi)\,ndw) + \sigma cMPS\frac{dq}{q} \end{aligned} \tag{A.63}$$

as claimed. Similarly, after using $MPN = MPCw\frac{u''(c)}{v''(n)}$, the second row of (A.61) is

$$\left[\begin{array}{cccc} \frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b} \end{array}\right] = MPN \left[\begin{array}{ccc} -\frac{q^2+V''(a)/u''(c)}{wV''(a)} & \frac{q}{V''(a)} & -1 \end{array}\right] \left[\begin{array}{cccc} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{array}\right] \tag{A.64}$$

Using (A.60) we find

$$-q\frac{u'(c)}{V''(a)}MPN = \frac{\sigma c}{q}q^2\frac{u''(c)}{V''(a)}MPC\left(\frac{-n\psi}{\sigma c}\right) = -\frac{n\psi}{q}MPS$$

Again the first column yields

$$\frac{\partial c}{\partial q} = -\frac{n\psi}{q}MPS - (a-b)\,MPN$$

The second and fourth column of (A.62) yield directly

$$\begin{aligned}\frac{\partial c}{\partial z} &= MPN \\ \frac{\partial c}{\partial b} &= qMPN\end{aligned}$$

Finally, since

$$\begin{aligned}\left(q^2+\frac{V''(a)}{u''(c)}\right)\frac{u'(c)}{V''(a)} &= -\sigma c\left(q^2\frac{u''(c)}{V''(a)}+1\right) \\ &= \psi n\frac{MPC}{MPN}\left(\frac{MPS}{MPC}+1\right)\end{aligned}$$

the third column yields

$$\frac{\partial n}{\partial w}=\frac{1}{w}\psi n\left(MPS+MPC\right)+MPNn=\frac{1}{w}\psi n\left(1+wMPN\right)+MPNn=\psi n\frac{1}{w}+MPN\left(n+\psi n\right)$$

The first-order total differential dn is then

$$\begin{aligned}dn &= \frac{\partial n}{\partial z}dz+\frac{\partial n}{\partial b}db+\frac{\partial n}{\partial q}dq+\frac{\partial n}{\partial w}dw \\ &= MPN\left(dz+qdb-(a-b)\,dq+(1+\psi)\,ndw\right)-\psi nMPS\frac{dq}{q}+\psi n\frac{dw}{w}\end{aligned} \tag{A.65}$$

□

Proof of theorem 2 in case 2a). If the policy functions are differentiable and the consumer is at an interior optimum, then the conditions of lemma A.1 are satisfied: the borrowing constraint is not binding so can be ignored, and the value function is concave per standard dynamic programming arguments. The notation of theorem 2 can be cast using that of the lemma by using the mapping

$$q\equiv Q \quad z\equiv y+\frac{\lambda}{\Pi} \quad a\equiv\lambda' \quad b\equiv\delta\frac{\lambda}{\Pi}$$

with $\frac{dP}{P}=\frac{d\Pi}{\Pi}$ and $\frac{dQ}{Q}=-\frac{dR}{R}$. Hence $dz=dy-\frac{\lambda}{\Pi}\frac{dP}{P}$, $db=-\delta\frac{\lambda}{\Pi}\frac{dP}{P}$ and $\frac{dq}{q}=-\frac{dR}{R}$; so

$$dz+qdb-(a-b)\,dq=dy-\underbrace{(1+Q\delta)\frac{\lambda}{\Pi}}_{\text{NNP}}\frac{dP}{P}+\underbrace{\left(\lambda'-\delta\frac{\lambda}{\Pi}\right)Q}_{\text{URE}}\frac{dR}{R}$$

Inserting this equation into (A.63) and (A.65) yields the desired result. □

Case 2b) $N>0$, no bonds, only R changing. Since we are not considering changes in wages, it is sufficient to restrict the analysis to a choice between consumption and assets. The following lemma then proves the result for dc. The result for dn follows as a straightforward extension.

Lemma A.2. *Let* $c(\theta, Y, R)$ *be the solution to the following consumer choice problem under concave preferences over current consumption* $u(c)$ *and assets* $W(\theta')$

$$\begin{array}{ll} \max_{c,\theta'} & u(c) + W(\theta') \\ \textit{s.t.} & c + (\theta' - \theta)\mathbf{S} = Y + \theta\mathbf{d} \end{array}$$

where $\frac{d\mathbf{S}}{dR} = -\frac{\mathbf{S}}{R}$. *Then, to first order*

$$dc = MPC\left(dY + URE\frac{dR}{R}\right) - \sigma(c)\,c\,(1 - MPC)\frac{dR}{R}$$

where $\sigma(c) \equiv -\frac{u'(c)}{cu''(c)}$ *is the local elasticity of intertemporal substitution,* $MPC = \frac{\partial c}{\partial Y}$*, and* $URE = Y + \theta\mathbf{d} - c$

Proof. The following first-order conditions characterize the solution

$$S^i u'(Y + \theta\mathbf{d} - (\theta' - \theta)\mathbf{S}) = W_{\theta^i}(\theta') \quad \forall i = 1\ldots N \tag{A.66}$$

Consider first an increase in income dY alone. Differentiating along (A.66) we find

$$S^i u''(c)\left(1 - \sum_j S^j \frac{d\theta'^j}{dY}\right) = \sum_j W_{\theta^i\theta^j}(\theta')\frac{d\theta'^j}{dY} \quad \forall i \tag{A.67}$$

Define $\eta^j \equiv S^j \frac{d\theta'^j}{dY}$. Then (A.67) rewrites

$$\sum_j \left(\frac{1}{S^i S^j} W_{\theta^i\theta^j}(\theta') + u''(c)\right)\eta^j = u''(c) \quad \forall i$$

Defining the matrix M with elements

$$m_{ij} \equiv \frac{1}{S^i S^j} W_{\theta^i\theta^j}(\theta') + u''(c)$$

this system can also be written in matrix form as

$$M\eta = u''(c)\,\mathbf{1}$$

or

$$\eta = u''(c)\,M^{-1}\mathbf{1}$$

The budget constraint then implies that

$$MPC = \frac{dc}{dY} = 1 - \sum_j \eta^j = 1 - u''(c)\,m \tag{A.68}$$

where m is defined as

$$m \equiv \mathbf{1}M^{-1}\mathbf{1} \tag{A.69}$$

Next, consider an increase in the real interest rate dR. Differentiating along (A.66) we now have

$$\frac{dS^i}{dR}u'(c)+S^iu''(c)\left(-\sum_j S^j\frac{d\theta'^j}{dR}-\sum_j\frac{dS^j}{dR}\left(\theta'^j-\theta^j\right)\right) = \sum_j W_{\theta^i\theta^j}(\theta')\frac{d\theta'^j}{dR} \quad \forall i$$

Using $\frac{dS^i}{S^i}=-\frac{dR}{R}$ this rewrites

$$-\frac{S^i}{R}u'(c)+S^iu''(c)\left(-\sum_j S^j\frac{d\theta'^j}{dR}+\sum_j\frac{S^j}{R}\left(\theta'^j-\theta^j\right)\right) = \sum_j W_{\theta^i\theta^j}(\theta')\frac{d\theta'^j}{dR} \quad \forall i \tag{A.70}$$

Defining now $\gamma^j\equiv S^j\frac{d\theta^{j\prime}}{dR}$, (A.70) shows that γ^j solves

$$\sum_j m_{ij}\gamma^j=-\frac{1}{R}u'(c)+u''(c)\sum_j\frac{S^j}{R}\left(\theta'^j-\theta^j\right) \quad \forall i$$

which rewrites in matrix form

$$M\gamma=\left(-\frac{1}{R}u'(c)+u''(c)\sum_j\frac{S^j}{R}\left(\theta'^j-\theta^j\right)\right)\mathbf{1}$$

or

$$\gamma=\left(-\frac{1}{R}u'(c)+u''(c)\sum_j\frac{S^j}{R}\left(\theta'^j-\theta^j\right)\right)M^{-1}\mathbf{1} \tag{A.71}$$

Differentiating with respect to R along the budget constraint $c=Y+\theta\mathbf{d}-(\theta'-\theta)\mathbf{S}$, we next see that

$$\frac{dc}{dR}=-\sum_j S^j\frac{\theta'^j}{dR}+\sum_j\frac{S^j}{R}\left(\theta^j-\theta'^j\right)=-\sum_j\gamma^j+\sum_j\frac{S^j}{R}\left(\theta^j-\theta'^j\right)$$

inserting (A.71) and using the definition of m,

$$\frac{dc}{dR} = -\left(-\frac{1}{R}u'(c)+u''(c)\sum_j\frac{S^j}{R}\left(\theta'^j-\theta^j\right)\right)m+\sum_j\frac{S^j}{R}\left(\theta^j-\theta'^j\right) \tag{A.72}$$

rearranging terms and using $u'(c)\equiv-c\sigma(c)u''(c)$ we find

$$\frac{dc}{dR} = -\sigma(c)\frac{c}{R}u''(c)m+\sum_j\frac{S^j}{R}\left(\theta'^j-\theta^j\right)(1-u''(c)m)$$

But using the expression for MPC in (A.68), this is simply

$$\frac{dc}{dR} = -\sigma(c)\frac{c}{R}(1-MPC)+\sum_j\frac{S^j}{R}\left(\theta^{j\prime}-\theta^j\right)MPC$$

and using the budget constraint $\sum_j S^j\left(\theta^{j\prime}-\theta^j\right)=(\theta'-\theta)\cdot\mathbf{S}_t=URE$ we obtain

$$\frac{dc}{dR}=-\sigma(c)\frac{c}{R}(1-MPC)+\frac{1}{R}URE\cdot MPC \tag{A.73}$$

Finally, considering a simultaneous change in income and the real interest rate, combining (A.68) and (A.73) we obtain the first order differential

$$dc = MPC\left(dY + URE\frac{dR}{R}\right) - \sigma(c)\,c\,(1 - MPC)\frac{dR}{R}$$

as was to be shown. □

A.7 Proof of Theorem 3

Given the assumption of fixed balance sheets and purely transitory shocks, Theorem 2 shows that

$$dc_i = M\hat{P}C_i\left(dY_i - dt_i + URE_i\frac{dR}{R} - NNP_i\frac{dP}{P}\right) - \sigma_i c_i\left(1 - M\hat{P}C_i\right)\frac{dR}{R}$$

where, where $dY_i = n_i e_i dw + we_i dn_i + d(d_i)$ is the change in gross income at the individual level and dt_i the change in taxes. We can further decompose the change in gross income as

$$dY_i = \frac{Y_i}{Y}dY + dY_i - \frac{Y_i}{Y}dY$$

and note that, since $\mathbb{E}_I[Y_i] = Y$,

$$\mathbb{E}_I\left[dY_i - \frac{Y_i}{Y}dY\right] = dY - \frac{\mathbb{E}_I[Y_i]}{Y}dY = 0 \tag{A.74}$$

Hence,

$$dc_i = M\hat{P}C_i\left(\frac{Y_i}{Y}dY + dY_i - \frac{Y_i}{Y}dY - dt_i + URE_i\frac{dR}{R} - NNP_i\frac{dP}{P}\right) - \sigma_i c_i\left(1 - M\hat{P}C_i\right)\frac{dR}{R}$$

and taking a cross-sectional average

$$\begin{aligned} dC \;=\; & \mathbb{E}_I\left[\frac{Y_i}{Y}M\hat{P}C_i\right]dY + \mathbb{E}_I\left[M\hat{P}C_i\left(dY_i - \frac{Y_i}{Y}dY\right)\right] \\ & -\mathbb{E}_I\left[M\hat{P}C_i\,(dt_i)\right] - \mathbb{E}_I\left[M\hat{P}C_i NNP_i\right]\frac{dP}{P} \\ & +\left(\mathbb{E}_I\left[M\hat{P}C_i URE_i\right] - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right)c_i\right]\right)\frac{dR}{R} \end{aligned} \tag{A.75}$$

Now, the government budget (13) with the fiscal rule $G_t = \overline{G}$ and target $\frac{B_t}{P_t} = \overline{b}$ reads

$$\mathbb{E}_I[t_{it}] = \overline{G} + \frac{B_t}{P_t} - \frac{\overline{b}}{R_t}$$

Using the fact that at the margin, taxes are adjusted lump-sum, and the fact that $NNP_g = -\overline{b}$ as well as $URE_g = -\frac{\overline{b}}{R}$, this implies

$$dt_i = dt = NNP_g\frac{dP}{P} - URE_g\frac{dR}{R}$$

In other words, taxes fall with unexpected increases in prices which reduce the government debt burden, and they fall with reductions in real interest rates which reduces the government's debt servicing costs. But the market clearing conditions (17) and (18) imply that these gains and losses have counterparts at the household level:

$$dt_i = dt = -\mathbb{E}_I\left[NNP_i\right]\frac{dP}{P} + \mathbb{E}_I\left[URE_i\right]\frac{dR}{R} \tag{A.76}$$

Hence, (A.75) rewrites

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y}M\hat{P}C_i\right]dY + \mathbb{E}_I\left[M\hat{P}C_i\left(dY_i - \frac{Y_i}{Y}dY\right)\right] \\ &\quad -\mathbb{E}_I\left[M\hat{P}C_i\right](dt) - \mathbb{E}_I\left[M\hat{P}C_i NNP_i\right]\frac{dP}{P} \\ &\quad +\left(\mathbb{E}_I\left[M\hat{P}C_i URE_i\right] - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right)c_i\right]\right)\frac{dR}{R} \end{aligned}$$

so

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y}M\hat{P}C_i\right]dY + \mathbb{E}_I\left[M\hat{P}C_i\left(dY_i - \frac{Y_i}{Y}dY\right)\right] \\ &\quad +\mathbb{E}_I\left[M\hat{P}C_i\right]\mathbb{E}_I\left[NNP_i\right]\frac{dP}{P} - \mathbb{E}_I\left[M\hat{P}C_i NNP_i\right]\frac{dP}{P} \\ &\quad +\left(\mathbb{E}_I\left[M\hat{P}C_i URE_i\right] - \mathbb{E}_I\left[M\hat{P}C_i\right]\mathbb{E}_I\left[URE_i\right] - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right)c_i\right]\right)\frac{dR}{R} \end{aligned}$$

and finally, using (A.74)

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y}M\hat{P}C_i\right]dY + \mathsf{Cov}_I\left(M\hat{P}C_i, dY_i - Y_i\frac{dY}{Y}\right) - \mathsf{Cov}_I\left(M\hat{P}C_i, NNP_i\right)\frac{dP}{P} \\ &\quad +\left(\mathsf{Cov}_I\left(M\hat{P}C_i, URE_i\right) - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right)c_i\right]\right)\frac{dR}{R} \end{aligned}$$

as claimed.

Case with heterogeneous taxes. If the taxes were not lump-sum, equation (A.76) would be replaced by

$$\mathbb{E}_I\left[dt_i\right] = -\mathbb{E}_I\left[NNP_i\right]\frac{dP}{P} + \mathbb{E}_I\left[URE_i\right]\frac{dR}{R}$$

we would therefore use the fact that

$$\mathbb{E}_I\left[M\hat{P}C_i\left(dt_i\right)\right] = \mathbb{E}_I\left[M\hat{P}C_i\right]\mathbb{E}_I\left[dt_i\right] + \mathsf{Cov}_I\left(M\hat{P}C_i, dt_i\right)$$

to finally obtain

$$\begin{aligned} dC &= \mathbb{E}_I\left[\frac{Y_i}{Y}M\hat{P}C_i\right]dY + \mathsf{Cov}_I\left(M\hat{P}C_i, dY_i - Y_i\frac{dY}{Y}\right) - \mathsf{Cov}_I\left(M\hat{P}C_i, NNP_i\right)\frac{dP}{P} \\ &\quad +\left(\mathsf{Cov}_I\left(M\hat{P}C_i, URE_i\right) - \mathbb{E}_I\left[\sigma_i\left(1 - M\hat{P}C_i\right)c_i\right]\right)\frac{dR}{R} - \mathsf{Cov}_I\left(M\hat{P}C_i, dt_i\right) \end{aligned}$$

The additional heterogeneous-taxation term is very natural. Suppose for example that, at the margin, gains from the government budget ($\mathbb{E}_I[dt_i] < 0$) lead to disproportionate reductions of taxes on high-MPC agents. Then $\mathrm{Cov}_I\left(M\hat{P}C_i, dt_i\right) < 0$, so aggregate consumption increases by more than the benchmark from Theorem 1. The opposite happens when tax reductions fall disproportionately on low-MPC agents.

A.8 Proof of corollary 2

From the definition of γ_i in (24), we have

$$d\left(\frac{Y_i}{Y}\right) = \gamma_i\left(\frac{Y_i}{Y}-1\right)\frac{dY}{Y}$$

Moreover,

$$dY_i - Y_i\frac{dY}{Y} = Yd\left(\frac{Y_i}{Y}\right) = \gamma_i\left(\frac{Y_i}{Y}-1\right)dY \tag{A.77}$$

Next, rewrite equation (19) in elasticity terms by dividing by per-capita consumption $C = \mathbb{E}_I[c_i]$ and using (A.77). We find

$$\begin{aligned}\frac{dC}{C} &= \underbrace{\mathbb{E}_I\left[\frac{Y_i}{\mathbb{E}_I[c_i]}M\hat{P}C_i\right]\frac{dY}{Y}}_{\text{Aggregate income channel}} + \underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \gamma_i\frac{Y_i}{\mathbb{E}_I[c_i]}\right)}_{\text{Earnings heterogeneity channel}}\frac{dY}{Y} - \underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{NNP_i}{\mathbb{E}_I[c_i]}\right)\frac{dP}{P}}_{\text{Fisher channel}} \\ &\quad + \left(\underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{URE_i}{\mathbb{E}_I[c_i]}\right)}_{\text{Interest rate exposure channel}} - \underbrace{\mathbb{E}_I\left[\sigma_i\left(1-M\hat{P}C_i\right)\frac{c_i}{\mathbb{E}_I[c_i]}\right]}_{\text{Substitution channel}}\right)\frac{dR}{R}\end{aligned}$$

Imposing $\gamma_i = \gamma$ and $\sigma_i = \sigma$ for all i, this equation writes

$$\begin{aligned}\frac{dC}{C} &= \underbrace{\mathbb{E}_I\left[\frac{Y_i}{\mathbb{E}_I[c_i]}M\hat{P}C_i\right]}_{\mathcal{M}}\frac{dY}{Y} + \gamma\times\underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{Y_i}{\mathbb{E}_I[c_i]}\right)}_{\mathcal{E}_Y}\frac{dY}{Y} - \underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{NNP_i}{\mathbb{E}_I[c_i]}\right)}_{\mathcal{E}_P}\frac{dP}{P} \\ &\quad + \left(\underbrace{\mathrm{Cov}_I\left(M\hat{P}C_i, \frac{URE_i}{\mathbb{E}_I[c_i]}\right)}_{\mathcal{E}_R} - \sigma\times\underbrace{\mathbb{E}_I\left[\left(1-M\hat{P}C_i\right)\frac{c_i}{\mathbb{E}_I[c_i]}\right]}_{\mathcal{S}}\right)\frac{dR}{R}\end{aligned}$$

which is equation (25).

B Data appendix

This section starts out by providing more details about the data and the MPC identification strategies for the SHIW (section B.1), the PSID (section B.2), and the CE (section B.3). Section B.4 contrasts the financial asset and liability information available in the PSID and the CE, and compares it to that available in the Survey of Consumer Finance (SCF).

Section B.5 performs a sensitivity analysis along several dimensions. Section B.5 considers the consequence of using total consumption expenditure to estimate MPC in the PSID and CE. Section B.5 considers the effect of excluding durable expenditures from URE. Section B.5 considers alternative maturity assumptions for assets and liabilities. Section B.5 considers robustness to the number of bins used to stratify the population in the PSID and in the CE. Finally, section B.5 considers alternative sample selections with respect to age in the PSID.

Section B.6 then cuts the data in various ways to examine the empirical drivers of the correlations in the data. Section B.6 looks at the influence of age, and section B.6 examines the role of income. Finally, section B.6 generalizes my covariance decomposition procedure from section 4.4 to multiple covariates and reports the decomposition when all of Jappelli and Pistaferri (2014)'s covariates are included.

B.1 SHIW

My first dataset comes from the 2010 wave of the Italian Survey of Household Income and Wealth, which is publicly available from the Bank of Italy's website. This is the data source employed by Jappelli and Pistaferri (2014), and it is very useful for my purposes because it contains a direct household-level measure of MPC, reported as part of a survey question.[47] An additional benefit of this dataset is that it presents detailed information on financial assets and liabilities, allowing a fairly precise measurement of URE and NNP for each household.

[47] "Imagine you unexpectedly receive a reimbursement equal to the amount your household earns in a month. How much of it would you save and how much would you spend? Please give the percentage you would save and the percentage you would spend."

Table B.1: Summary statistics in the SHIW

	N	mean	p5	p25	p50	p75	p95
Income	7,951	36,114	9,565	19,857	30,719	45,340	81,320
Consumption	7,951	27,541	10,600	16,800	24,000	32,900	56,500
Maturing assets	7,951	27,073	0	2,000	10,000	30,000	97,929
Maturing liabilities	7,951	9,440	0	0	0	305	49,000
URE	7,951	26,207	-24,787	2,490	16,214	39,063	113,834
Nominal assets	7,951	22,499	0	1,274	6,796	22,000	77,272
Nominal liabilities	7,951	15,133	0	0	0	4,285	99,000
Net Nominal Position	7,951	7,366	-81,712	-1	3,830	17,115	71,218
MPC	7,951	0.47	0.00	0.20	0.50	0.80	1.00

Units: 2010 Euros. All statistics are computed using survey weights.

Exposure measures

Following the structure of the survey, I measure all my statistics at an annual frequency.[48] Table B.1 presents summary statistics in euros.

URE: $Y - T - C + A - L$**.** To construct my measure of unhedged interest rate exposure, I use net annual disposable income (which includes taxes, transfers and capital returns) as my measure of income net of taxes $Y - T$. My consumption measure C includes expenditures on both durables and non durables goods, but it does not include interest payments, which are not reported separately from principal payments, and which I therefore count as part of L.

For assets maturing in the year (A), I consider the amounts held in checking accounts, savings accounts, certificates of deposits, and repurchase agreements (maturing equities and bonds are already included as the dividend income part of $Y - T$). I consider various scenarios for maturities. Given an assumed maturity of N_j years for a given asset or liability j, I scale the observed amounts by $\frac{1}{N_j}$ to obtain an annual measure of maturing flows. In my benchmark scenario, I assume that these assets have a duration of two quarters $(N_j = \frac{1}{2})$.

[48] Note that the time frame for MPC is not specified in the question, as issue that is left unresolved in Jappelli and Pistaferri (2014). A follow-up question in the 2012 SHIW separates durable and nondurable consumption, and specifies the time frame as a full year. The equivalent "MPC" out of both durable and nondurable consumption has close to the same distribution as that of MPC in the 2010 SHIW (respective means are 47 in 2010 and 45 in 2010) which suggests that households tended to assume that the question referred to the full year.

As part of liabilities maturing in the year (L), I include payments on all loans. The SHIW records up to three mortgages for each households. I add to principal payments the principal *balance* outstanding on adjustable rate mortgages, assuming a duration of three quarters ($N_j = \frac{3}{4}$). I also include all debt outstanding on credit cards, assuming a duration of two quarters.

Section B performs a sensitivity analysis around these maturity assumptions.

NNP and Income. To construct my measure of net nominal position, I include in nominal assets the full amount held in checking accounts, savings accounts, certificates of deposits and repurchase agreements. I also include the full amounts held in bonds from Italian banks and firms, with the exception of inflation-indexed BTP bonds. I assume that two-thirds of foreign bonds are denominated in euros, and count that amount in nominal assets. I then include all the shares of money market mutual funds and bonds mutual funds, in keeping with Doepke and Schneider (2006). For shares held at 'mixed' mutual funds, I assume that half of those are indirectly invested in bonds. Finally, I count all credit originating from commercials or private party loans.

For nominal liabilities, my measure includes all debt due to banks, other financial institutions, and other households, as well as commercial loans.

My results are not influenced in any meaningful way by altering the share of 'mixed' mutual funds invested in bonds, the share of foreign bonds that are euro-denominated, or by excluding commercials and private party loans from both nominal assets and liabilities.

For my income exposure measure Y, since the SHIW does not provide information on government taxes and transfers, I assume that the exposure is based on net rather than gross income. Other assumptions, such as assuming a constant tax rate, only have a minor influence on the size of the relevant covariance.

B.2 Panel Study of Income Dynamics

For the Panel Study of Income Dynamics website at the University of Michigan, I assemble a dataset with household-level information on consumption, income, assets and liabilities. This base file, together with a data dictionary, is included in the replication folder. The procedure to identify MPC out of transitory income shocks that I employ in this section originates from the contribution of Blundell et al. (2008) (BPP). It has been used by Kaplan et al. (2014) to estimate the MPCs of hand-to-mouth households, and by Berger et al. (2015) to estimate MPCs at different levels of housing wealth. My sample selection closely follows these papers. Since the PSID only starts recording detailed consumption information in 1999, my sample period starts with the 1999 wave, and ends in 2013. I use the core sample of the PSID (made up of the SCR, SEO and Immigrant samples) and drop households with missing information on the head's race, education or the state of residence. I then drop households whose income or consumption grows more than 500%, falls by more than 80%, or is below $100 in any period. I treat top-coded income or consumption data as missing data.

While the literature usually restricts the sample to working-age households, in my benchmark scenario I keep all families whose head is between 20 and 90 years old, in order to have a more accurate picture of the cross-sectional distribution of UREs and NNPs by age.[49] This sample selection leaves me with 38,143 observations from 9,620 different households.

Exposure measures

Just as for the SHIW, I follow the structure of the PSID survey and do all my measurement at annual frequency. The PSID groups assets and liabilities into coarse categories, so I sometimes need to take a stand on their internal composition. I deflate all monetary variables to 2009 dollars using the CPI in order to ensure comparability over time. Table B.2 reports summary statistics in 2009 dollars.

URE: $Y - T - C + A - L$**.** For URE, I use an annual measure of net disposable income for $Y - T$ (which includes capital returns), and an annual consumption measure C that includes only the consumption categories continuously available in the survey since 1999 (my first sample year). Those consists of expenditures on food, rent, property taxes, home insurance, utilities, telecommunications, trans-

[49] As we know from Doepke and Schneider (2006), young and old households tend to have the largest net nominal positions, with opposite signs. Since households' income processes tend to change upon entering retirement, however, including older households could lead to noisier estimates of MPCs. For this reason, in section B.5 I provide results for my elasticity estimates that restrict the PSID sample to households aged 25 to 55 years old, as is common in the literature.

Table B.2: Summary statistics in the PSID

	N	mean	p5	p25	p50	p75	p95
Net Income	38,143	60,753	13,722	29,546	47,914	74,659	138,136
Consumption	38,143	28,556	9,640	16,937	24,604	34,939	60,662
Maturing assets	38,143	41,639	0	828	6,593	25,761	175,813
Maturing liabilities	38,143	23,008	0	2,289	10,308	22,638	72,213
URE	38,143	50,828	-40,105	1,642	18,945	53,658	225,344
Nominal assets	38,143	40,221	0	600	5,000	25,761	185,000
Nominal liabilities	38,143	77,546	0	382	27,632	118,499	292,862
Net Nominal Position	38,143	-37,324	-260,753	-94,686	-11,973	1,740	127,464
Pre-govtt income	38,143	76,302	8,912	31,045	56,961	94,577	187,254

Units: 2009 USD. All statistics are computed using survey weights.

portations, education, childcare and healthcare. Loan repayments are included in L, since — just like in the SHIW — interest expenses are not reported separately from principal payments.

For assets maturing in the year (A), the PSID contains a variable that groups together checking accounts, saving accounts, money market mutual funds, certificates of deposit, government savings bonds and T–bills. In my benchmark scenario, I assume a duration of two quarters for this asset category.

For the remainder of liabilities (L), the PSID reports up to two mortgages for each household. In my benchmark scenario I assume that the duration for ARMs is three quarters. The PSID also contains a variable that includes credit cards debt, student loans, medical bills, legal debt and loan from relatives. From 2011 onwards, a breakdown of categories is available, and credit cards account for an average of 40% of the total. I assume that this fraction has been constant over time, and maintain my assumption of two quarter duration for credit card debt.

NNP and Income. To construct a household's net nominal position, I count as nominal assets all the amount held in checking accounts, saving accounts, money market mutual funds, certificates of deposit, government savings bonds and T–bills. The PSID contains another variable that includes bonds, trusts, estates, cash value of life insurance and collection. I assume that half of that is constituted by bonds, and include this in nominal assets as well. I include the whole amount in IRAs invested in bonds, and half the amount in IRAs invested in a mix of stocks and bonds.

For nominal liabilities, I count the principal balance outstanding on each mortgage and the whole amount due in the form of credit cards debt, student loans, medical bills, legal debt and loan from relatives.

For my income exposure measure, I use the PSID's measure of gross income before taxes and government transfers.

Identification of MPC

As mentioned in main text, the literature exploits the panel dimension of the data in PSID in order to estimate the MPC out of transitory income shocks. I follow BPP and construct my consumption measure for MPC using all non durable consumption categories.[50] For my income measure, I use labor income plus government transfers, as in Kaplan et al. (2014). Following BPP, I first regress the log of consumption and the log of income on observables characteristics of the households, including dummy variables for year, year of birth, education, race, family structure, employment status and region, as well as dummies for interactions between year with education, race, employment status and region. I then use the residuals of these regressions (call them y_{it} and c_{it}) to estimate the MPC out of transitory income shocks. Specifically, for each exposure measure, in each year, I stratify the population in J bins. I then estimate $\psi_j = \frac{\text{Cov}_j(\triangle c_t, \triangle y_{t+1})}{\text{Cov}_j(\triangle y_t, \triangle y_{t+1})}$ as the pass-through coefficient of log income on log consumption, pooling all years together.[51] I finally recover a measure of the marginal propensity to consume MPC_j by multiplying ψ_j by the ratio of average consumption to average income in each bin j.

Next, for each exposure measure, I calculate the average value of exposure in each bin, EXP_j, normalized by average consumption in the sample. I finally compute my estimators as[52]

[50] This is also consistent with Kaplan et al. (2014) and Berger et al. (2015). In section B.5, I report instead an MPC calculated using all consumption expenditures available in the PSID.

[51] See Blundell et al. (2008) and Kaplan et al. (2014) for the structural assumptions under which this procedure correctly recovers the MPC out of transitory income shocks. The estimate can, of course, be recovered with an instrumental variable regression of $\triangle c_t$ on $\triangle y_t$, using $\triangle y_{t+1}$ as an instrument.

[52] Note that I simply take $\widehat{S}$ to be the sample counterpart to $1 - \mathbb{E}_I[MPC]$. The procedure cannot simultaneously recover an estimate of the covariance between MPC and consumption. In the SHIW data, the difference between average MPC and consumption-weighted MPC is small, so this is unlikely to significantly affect the value of S.

$$\widehat{\mathcal{E}_{EXP}^{NR}} = \frac{1}{J}\sum_{j=1}^{J} MPC_j EXP_j$$

$$\widehat{\mathcal{E}_{EXP}} = \widehat{\mathcal{E}_{EXP}^{NR}} - \left(\frac{1}{J}\sum_{j=1}^{J} MPC_j\right)\left(\frac{1}{J}\sum_{j=1}^{J} EXP_j\right)$$

$$\widehat{S} = 1 - \left(\frac{1}{J}\sum_{j=1}^{J} MPC_j\right)$$

In order to take into account sampling uncertainty, I compute the distribution of these estimators using a Monte-Carlo procedure, resampling the panel at the household level with replacement. Section B.5 considers robustness to using $J = 3$ to 8 bins to stratify the sample.

B.3 Consumer Expenditure Survey, 2001-2002 (JPS sample)

My data for the Consumer Expenditure Survey comes from the Johnson et al. (2006) (JPS) dataset, which I merged with the main survey data and detailed expenditure files to obtain additional information on households's consumption expenditures, financial assets and liabilities. The dataset covers households with interviews between February 2001 and March 2002. Relative to the full CE sample, JPS drop the bottom 1% of nondurable expenditure in levels, households living in student housing, those with age less than 21 or greater than 85, those with age changing by more than a unit or by a negative amount between quarters, and those whose family size changes by more than three members between quarters. Since the 2001 CE survey has several observations with missing values for income — which is a crucial component of URE and a measure of exposure in its own right — I do not consider observations with incomplete income information when analyzing the interest rate exposure or the earnings heterogeneity channel. My sample is therefore made of 9,983 observations from 4,833 different households when computing statistics relevant to these two channels, and contains 12,227 observations from 5,900 households when analyzing the Fisher channel.

Exposure measures

Following the structure of the dataset, all my CE measurement is performed at a quarterly frequency. Table B.3 presents summary statistics in dollars.

Table B.3: Summary statistics in the CE

	N	mean	p5	p25	p50	p75	p95
Net income	9,983	11,621	1,445	4,550	8,995	15,643	29,706
Consumption	9,983	9,993	2,187	4,633	7,625	12,514	26,205
Maturing assets	9,983	4,796	0	0	60	1,769	21,000
Maturing liabilities	9,983	5,334	0	0	782	2,954	35,502
URE	9,983	1,582	-28,561	-2,874	596	4,916	26,890
Nominal assets	12,227	19,006	0	0	9	5,000	100,000
Nominal liabilities	12,227	49,671	0	0	12,786	73,951	200,794
Net Nominal Position	12,227	-27,859	-174,318	-58,441	-6,801	0	58,078
Pre govt. income	9,983	12,520	1,731	4,814	9,500	16,750	32,500

Units: 2001 USD. All statistics are computed using survey weights.

URE: $Y - T - C + A - L$**.** In order to construct my quarterly measure of URE, I use one fourth of the annual net disposable income as my measure of income $Y - T$, while for consumption C I use a quarterly measure of total expenditures that include both durables and non durables goods.

For maturing financial assets A, as in the other two surveys, I assume that checking accounts and savings accounts have a duration of two quarters.

For maturing liabilities, as in the SHIW and PSID, my benchmark assumptions is that the duration of an adjustable rate mortgages is three quarters, and that credit card debt has a two quarter duration. Relative to those surveys, the CE also contains information on adjustable-rate home equity loans, for which I also assume a three quarter duration. As before, I also include all principal payments carried out in the period towards my measure of L.

NNP and Income. To construct my NNP measure, I include in nominal assets all the amount in savings and checking accounts. I then assume that the variable 'securities', which contains the amount held in stocks, mutual funds, private sector bonds, government bonds or Treasury notes, contains a 50% share of bonds, and include those in my measurement. I also count all the amount held in US savings bonds and in private party loans owed.

Using the supplemental expenditure files, my measure of nominal liabilities is fairly detailed. I take the sum of principal balances outstanding on mortgages, home equity loans, home equity line of credit, loans on vehicles, personal debt and credit card debt.

For my income exposure measure, I use an annual measure of gross income before taxes, converted to quarterly value.

MPC identification strategy

JPS identified the propensity to consume out of the 2001 tax rebate by exploiting random variation in the timing of its receipt across households. In this section I closely follow their procedure for analyzing responses to the rebate among different exposure groups. Specifically, for each of my redistribution channels, I rank households in equally-sized bins according to their measure of exposure as at the time of the first interview. I then regress changes in the level of consumption expenditures (ΔC_{it} in JPS's notation) on the amount of the tax rebate ($Rebate_{it}$). I follow their instrumental-variable specification, instrumenting $Rebate_{it}$ with a dummy indicator for whether the debate was received. I include month effects and control for age and changes in family composition, and I allow both the intercept and the rebate coefficients to differ across households bins.

My benchmark estimate uses food consumption expenditures as dependent variable. This allows for substantially more precise estimates, as it does in JPS. Section B.5 below reports all results using total consumption expenditures as dependent variable instead. Section B.5 considers using different numbers of bins.

The procedure to compute estimators is then the same one as the PSID, with confidence intervals again constructed using a Monte-Carlo procedure, resampling the panel at the household level with replacement. Section B.5 reports redistribution elasticities using between 3 and 8 bins to stratify the sample.

B.4 Evaluating the quality of the financial information in U.S. surveys

In order to shed light on the quality of financial data in the PSID and the CE, tables B.4 and B.5 compare the median value of each class of assets and liabilities for households holding these instruments with the comparable number from the Survey of Consumer Finance. All three surveys are analyzed in 2001, the year in which they all overlap. As discussed above, the CE and the PSID group assets and liabilities into coarse categories, making a precise comparison difficult. However, table B.4 illustrates that liabilities in both the CE and the PSID appear to be aligned with numbers from the SCF as far as medians are concerned. This is especially true in the CE. Regarding financial assets, PSID and SCF data are fairly comparable. By contrast, the CE appears to considerably underreport assets, confirming claims in the literature.

Table B.4: Median values for financial liabilities — CE v. PSID v. SCF

Liabilities	SCF	CE	PSID	CE/SCF	PSID/SCF
Mortgages on primary residence	72	72.3	78	1.00	1.08
HELOC on primary residence	15	18.9	-	1.26	-
Other residential debt	40	37.9	18	0.95	0.45
Credit cards	1.9	2		1.05	
Vehicle loans	9.2	10.4	6	1.13	0.6
Education loans, personal loans, other	5	1.2		0.24	
Any debt	38.7	40.1	50	1.04	1.29

Units: Thousands of 2001 USD.
Households holding those liabilities in 2001. Medians computed using survey weights.

Table B.5: Median values for financial assets — CE v. PSID v. SCF

Financial Assets	SCF	CE	PSID	CE/SCF	PSID/SCF
Transaction accounts	3.9	1		0.26	
Certificates of deposit	15	3	3.5	0.2	0.7
Savings bonds	1	0.8		0.8	
Retirement accounts	29.4	-	25	-	0.85
Stocks	20	25	20	0.64	1
Bonds, mutual funds, life insurance, other	20		12		0.6
Any financial asset	28.3	4.5	8	0.16	0.28

Units: Thousands of 2001 USD.
Households holding those assets in 2001. Medians computed using survey weights.

B.5 Sensitivity analysis

In this section I perform several robustness checks. As a general matter, my results in the SHIW and PSID are remarkably stable across all scenarios.

Using total expenditure to estimate MPC

Figure B.1 replicates the right two columns of figure 2 when *all* available consumption expenditures are used to estimate MPC in the PSID and in the CE, instead of my benchmark scenario (which uses non durable consumption in the PSID and food consumption in the CE). This involves a minor change of consumption measure for the PSID, but a much more substantial change in the CE.

As a result, for the PSID, figure B.1 delivers qualitatively similar patterns as figure 2. Conversely, for the CE, the patterns are different, but the confidence intervals are extremely wide, and the point estimates tend to give implausible values, either very close to 1 or below 0.

Table B.6 replicates the right two columns of table 3 with this alternative definition of MPC. In the PSID, the point estimates are little changed, though confidence intervals are larger. In the CE, by contrast, the signs are the same, but the magnitudes are larger than in my benchmark scenario. However, the confidence bands are very wide. Moreover, income-weighted MPC is now negative on average, though again with very large confidence intervals. I conclude that this measure of MPC, while theoretically more appealing, is too imprecise to be able to draw definitive conclusions.

Excluding durable consumption from the URE calculation

Section 2.2 shows that, if relative durable goods prices have an elasticity ϵ with respect to the real interest rate, then a theoretically-consistent measure of URE counts a fraction $1-\epsilon$ of nondurable expenditures. Figure B.2 plots my estimated $\widehat{\mathcal{E}_R}$ against ϵ in all three datasets. The left-most part of the graph corresponds to $\epsilon = 0$, which is my benchmark scenario. As is clear from the graphs, the magnitudes are not altered dramatically by the choice of ϵ. If anything, excluding a larger fraction of durable goods tends to make the estimated value of $\mathcal{E}_R$ more negative.

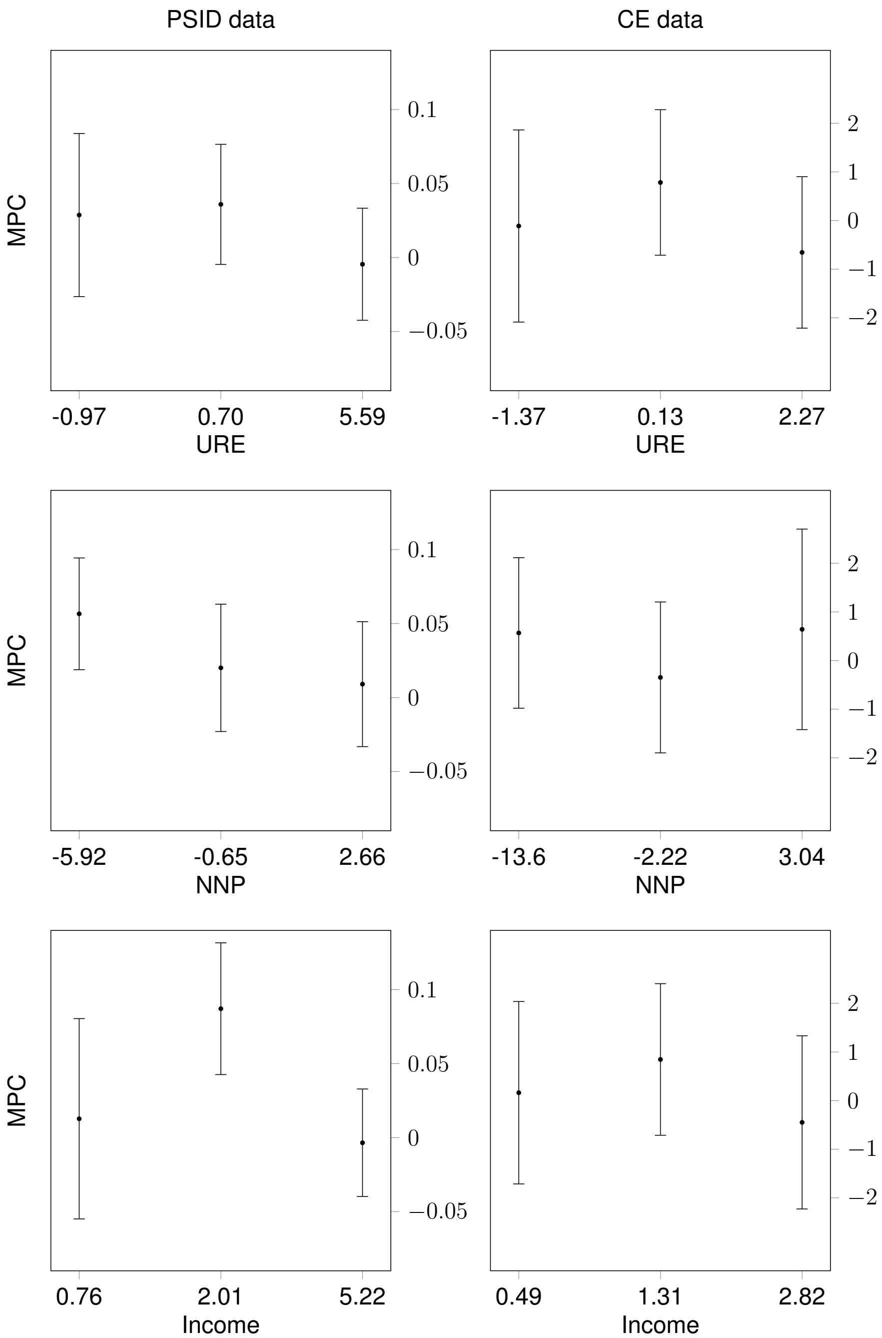

Figure B.1: Using total expenditures to estimate MPC in the PSID and the CE

Alternative maturity assumptions

Here, I consider the sensitivity of my estimates of the redistribution elasticity with respect to the real interest rate, $\mathcal{E}_R$, to my assumptions regarding maturities for short-term assets and liabilities that I am counting as part of A_i or L_i. Table B.7 reports results. In the first column, I assume that all assets and liabilities have a duration of one quarter. In the second, I maintain these assumptions, but increase

Table B.6: Using total expenditures to estimate MPC in the PSID and CE

Survey	PSID		CE	
	Estimate	95% C.I.	Estimate	95% C.I.
$\widehat{\mathcal{E}_R}$	-0.04	[-0.12,0.03]	-0.41	[-2.00,1.17]
$\widehat{\mathcal{E}_R^{NR}}$	-0.01	[-0.08,0.06]	-0.41	[-1.98,1.16]
$\widehat{S}$	0.98	[0.95,1.01]	0.99	[-0.02,2.01]
$\widehat{\mathcal{E}_P}$	-0.07	[-0.15,0.01]	-0.44	[-7.03,6.15]
$\widehat{\mathcal{E}_P^{NR}}$	-0.11	[-0.19,-0.03]	-1.67	[-9.13,5.80]
$\widehat{\mathcal{M}}$	0.06	[-0.01,0.12]	-0.03	[-1.93,1.88]
$\widehat{\mathcal{E}_Y}$	-0.03	[-0.08,0.02]	-0.31	[-1.37,0.74]

This figure recomputes the right two columns of table 3, but uses total expenditures to estimate MPC.

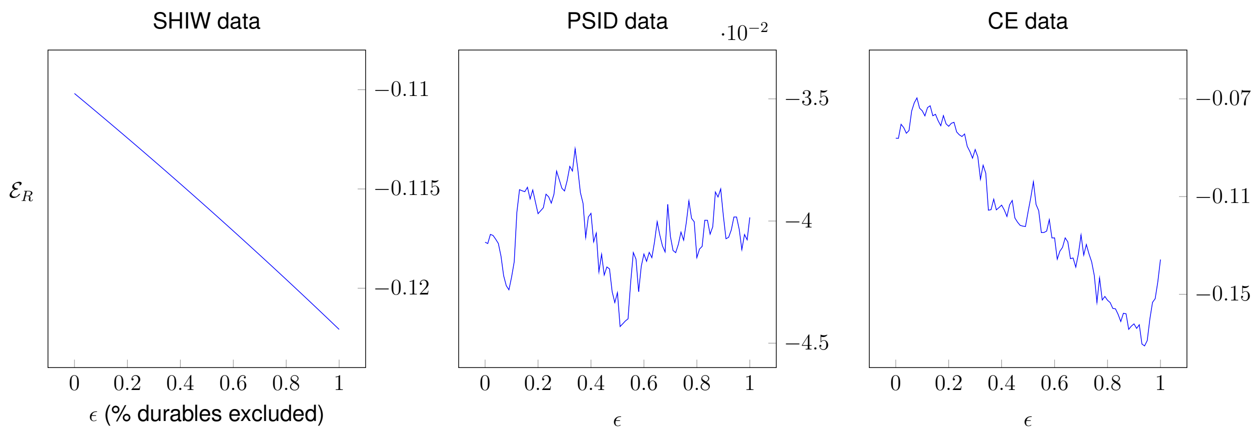

Figure B.2: Estimating $\widehat{\mathcal{E}_R}$ assuming alternative values of ϵ.

ARM mortgage durations to two quarters. The third column is my benchmark scenario (two quarter duration for deposits, three for ARMs, two for credit card debt). My fourth scenario increases the duration of deposits to three quarters, the ARM durations to one year, and credit card debt durations to three quarters. Finally, the last column reports results assuming durations of one year.

A stylized fact emerging from these results is that, the longer the durations, the closer to zero $\widehat{\mathcal{E}_R}$ becomes. This is consistent with the predictions from my model in section 5, and in particular the left panel of figure 3. The point estimates do not vary dramatically across scenarios, and remain negative in all scenarios across all three datasets, suggesting that my benchmark estimates are robust to maturity assumptions.

Table B.7: Estimated redistribution elasticity $\mathcal{E}_R$ for five duration scenarios

		Duration scenario				
		Quarterly	Short	Benchmark	Long	Annual
$\widehat{\mathcal{E}_R}$	SHIW	-0.18 [-0.29,-0.06]	-0.21 [-0.29,-0.13]	-0.11 [-0.16,-0.06]	-0.08 [-0.11,-0.04]	-0.06 [-0.09,-0.03]
	PSID	-0.10 [-0.20,-0.00]	-0.10 [-0.19,-0.01]	-0.05 [-0.10,-0.00]	-0.04 [-0.09,0.00]	-0.03 [-0.07,0.01]
	CE	-0.19 [-0.55,0.17]	-0.13 [-0.41,0.15]	-0.09 [-0.26,0.09]	-0.07 [-0.20,0.06]	-0.06 [-0.17,0.06]

Table B.8: Redistribution elasticities using 3 to 8 bins in the PSID and the CE

		Number of bins					
		3	4	5	6	7	8
PSID	$\widehat{\mathcal{E}_R}$	-0.05 [-0.10,-0.00]	-0.04 [-0.09,0.02]	-0.04 [-0.10,0.02]	-0.05 [-0.11,0.01]	-0.05 [-0.12,0.01]	-0.05 [-0.11,0.01]
	$\widehat{\mathcal{E}_P}$	-0.02 [-0.08,0.04]	-0.04 [-0.10,0.02]	-0.03 [-0.10,0.03]	-0.02 [-0.08,0.04]	-0.03 [-0.10,0.04]	-0.03 [-0.09,0.04]
	$\widehat{\mathcal{E}_Y}$	-0.04 [-0.08,-0.00]	-0.04 [-0.08,-0.00]	-0.05 [-0.09,-0.01]	-0.05 [-0.09,-0.00]	-0.05 [-0.09,-0.00]	-0.05 [-0.09,0.00]
CE	$\widehat{\mathcal{E}_R}$	-0.09 [-0.26,0.09]	-0.10 [-0.29,0.09]	-0.14 [-0.36,0.08]	-0.10 [-0.33,0.14]	-0.12 [-0.37,0.13]	-0.10 [-0.35,0.16]
	$\widehat{\mathcal{E}_P}$	-0.11 [-0.83,0.60]	-0.18 [-0.93,0.58]	-0.28 [-1.17,0.61]	-0.20 [-1.18,0.78]	-0.42 [-1.50,0.66]	-0.60 [-1.66,0.46]
	$\widehat{\mathcal{E}_Y}$	-0.05 [-0.15,0.06]	-0.04 [-0.14,0.06]	-0.06 [-0.17,0.05]	-0.05 [-0.17,0.07]	-0.09 [-0.21,0.03]	-0.06 [-0.19,0.06]

Number of bins in the PSID and CE

Recall that my estimates of MPCs in the PSID and the CE are obtained by I stratifying the population in three equally-sized groups. Table B.8 reports the full redistribution elasticities of all three channels by progressively increasing the number of bins from 3 to 8 bin in both samples.

In the PSID results are fairly robust, both in terms of point estimates (which remaining negative and close to benchmark-scenario values) and in terms of confidence intervals (which remain relatively narrow). Signs are also stable in the CE, though for $\widehat{\mathcal{E}_P}$ magnitudes increase and confidence bands also widen considerably.

Table B.9: Sensitivity with respect to age sample selection in PSID

Survey	Benchmark		[25,55]	
	Estimate	95% C.I.	Estimate	95% C.I.
$\widehat{\mathcal{E}_R}$	-0.05	[-0.10,-0.00]	-0.03	[-0.08,0.01]
$\widehat{\mathcal{E}_R^{NR}}$	0.01	[-0.05,0.06]	0.03	[-0.01,0.07]
$\widehat{S}$	0.97	[0.95,0.98]	0.96	[0.94,0.98]
$\widehat{\mathcal{E}_P}$	-0.02	[-0.08,0.04]	-0.02	[-0.08,0.04]
$\widehat{\mathcal{E}_P^{NR}}$	-0.07	[-0.13,-0.01]	-0.10	[-0.17,-0.03]
$\widehat{\mathcal{M}}$	0.08	[0.03,0.13]	0.10	[0.04,0.15]
$\widehat{\mathcal{E}_Y}$	-0.04	[-0.08,-0.00]	-0.04	[-0.07,-0.00]

Age sample selection in PSID

Table B.9 assesses the impact of including very young and very old households in my baseline PSID sample, which departs from the benchmark assumption in the literature. The first set of columns recall my benchmark estimates for all my key cross sectional moments. The second one reports results obtained when restricting the sample to households whose head is between 25 and 55 years old, as in Kaplan et al. (2014) and others. As evident from the table, results are broadly consistent for both samples, suggesting that age is not an important driver of my results.

B.6 Empirical drivers of MPC, URE, NNP and income

This section complements section 4.4 by providing other perspectives on the empirical drivers of my main objects of interest.

The role of age

This section examines the distribution of exposures and MPC by age in each survey. I divide the population in eight equally-sized age bins. This allows me to assess life-cycle dynamics. It also helps to visualize clearly the relative strengths and weaknesses of each survey.

Exposure measures. Figure B.3 reports the average value of URE, NNP and income in each age bin, normalized by average consumption in the survey. Average URE (the blue line in the first row of graphs) is increasing in age across all three surveys, with a pattern of decline after retirement in the SHIW. This pattern is mostly due to a decumulation of financial assets in that survey (as represented by the green line). In terms of magnitudes, average URE is always positive in the SHIW and in the PSID, while in the CE average URE is negative for most working-age households. However, this is clearly driven by the different data flaws in each survey: the SHIW and the PSID greatly underreport consumption relative to income — notice the difference between the black and the red line. This tends to overestimate URE. By contrast, as documented above, the CE severely underreports assets, underestimating URE.

Regarding net nominal positions (the blue line in the second row of graphs), the life-cycle pattern in the SHIW is also increasing in age. By contrast, the PSID and the CE display an interesting U shape, with a minimum around age 40. In particular, in the SHIW, nominal liabilities are declining almost monotonically with age, while nominal assets are sharply increasing until age 60 and then decline rapidly. By contrast, in the PSID and in the CE, nominal liabilities are increasing in age for young households, and then start to decline steadily after age 40 — while nominal assets are almost monotonically increasing in age. In terms of magnitudes, average NNP is negative for most of working age population in the SHIW, while it is very negative in the CE and PSID for all households cohorts except the oldest ones. This highlights, once again, the issue that these surveys cover liabilities better than they cover assets.

For income, we observe the classic inverted-U shape in age across all three datasets.

MPC

Figure B.4 reports marginal propensities to consume by age bins in all three surveys. There is an overall declining pattern in age, except for a spike for the oldest cohort in the CE. Interestingly, all three surveys also suggest a rise in MPC around middle age. This pattern is not sensitive to the number of bins employed to stratify the population. Combining this graph with figure B.3, it appears that age is indeed a driver of the negative correlation between MPC and my exposure measures — as already apparent in table 4.

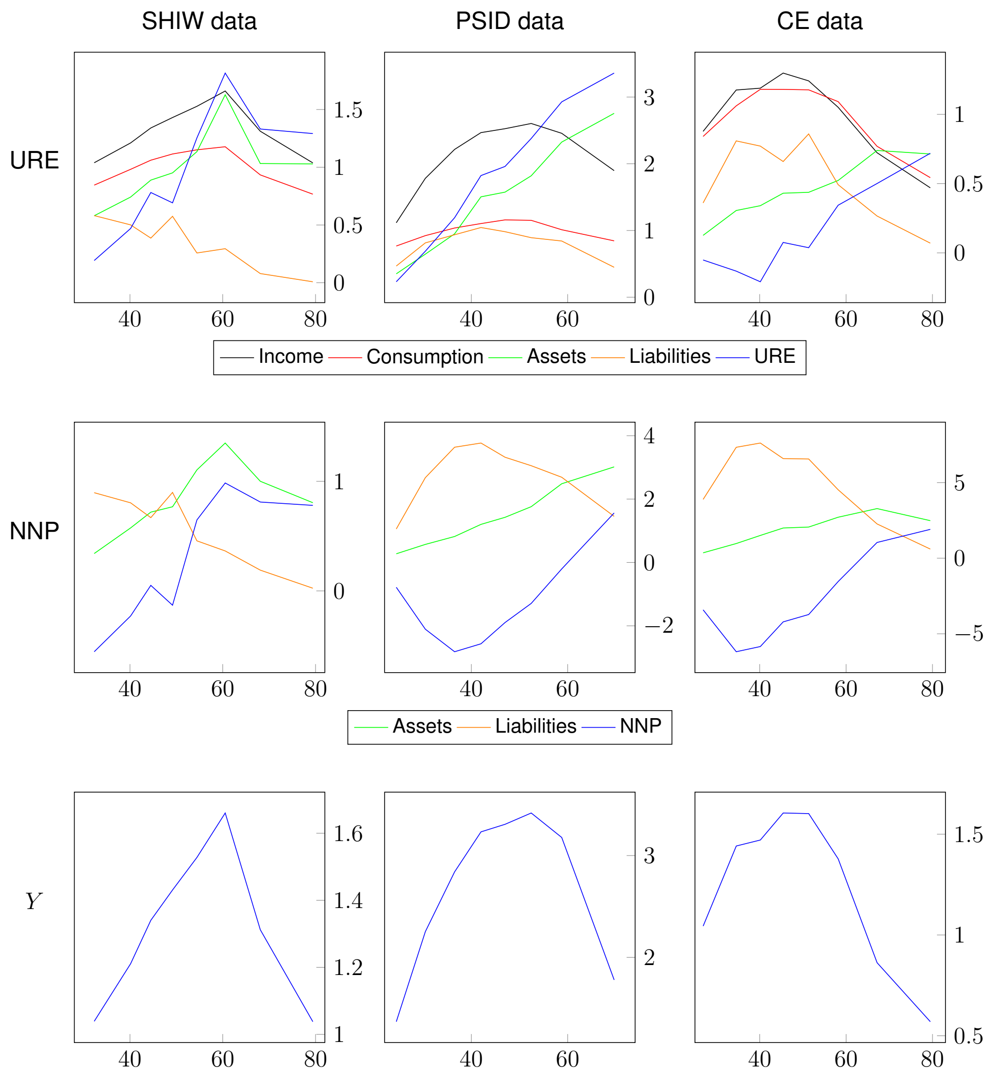

Figure B.3: Exposure measures by age bins in all three datasets

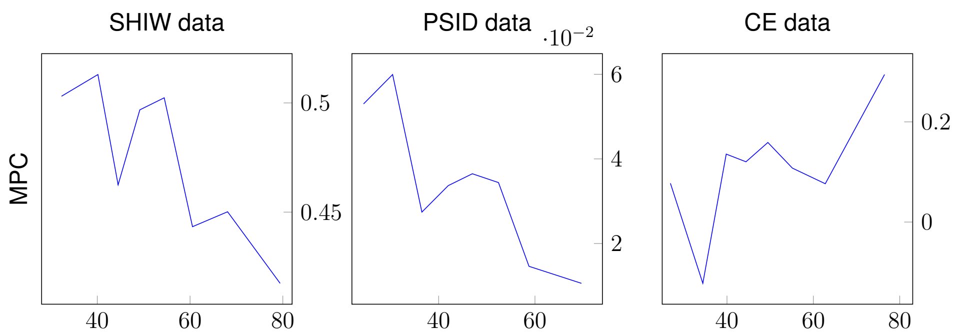

Figure B.4: MPCs by age bins in all three datasets

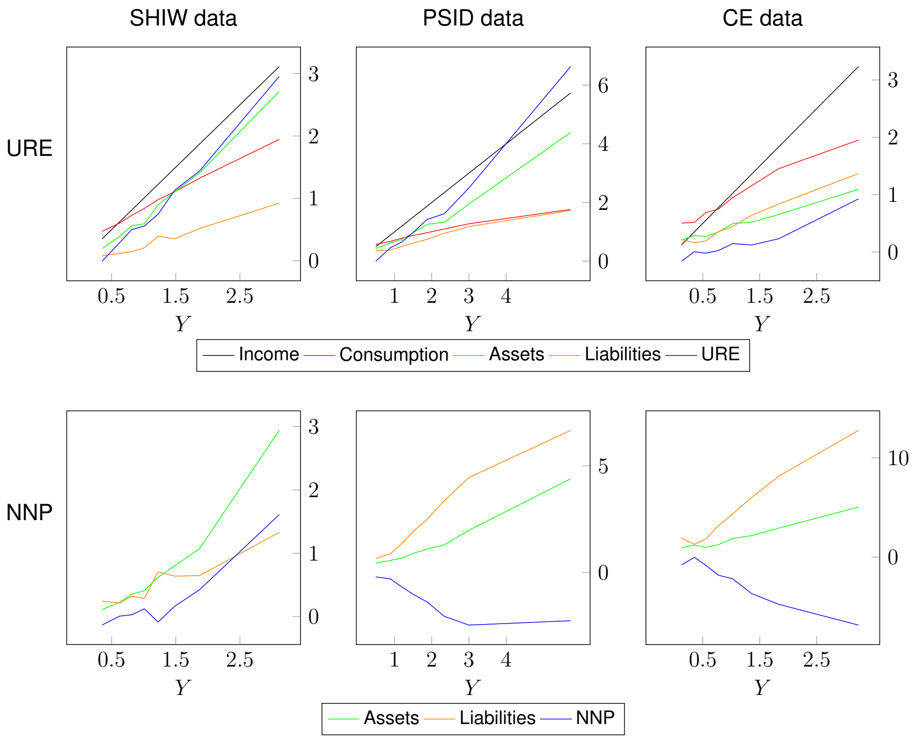

Figure B.5: URE and NNP components by income bins in all three datasets

The role of income

Figure B.5 examine the distribution of URE and NNP in all three surveys, when the population is grouped into eight income bins. Unsurprisingly, average URE is increasing in income, especially in the SHIW and the PSID. In these surveys, average URE increases more than one for one with income at the top of the distribution, owing an increase in maturing assets. Interestingly, maturing liabilities (the orange line) also increase in income across all three surveys.

For net nominal position, patterns are different in Italy and in the United States. In the SHIW, net nominal position is initially flat, and then increases with income, owing to an increase in assets at the top of the income distribution. By contrast, in the PSID and in the CE, net nominal position initially declines in income, and then flattens out. This is because nominal liabilities initially increase strongly with income, while nominal assets only increase mildly.

In section 4.4, I presented a covariance decomposition that projected observables on a single covariate. This approach can of course be generalized to include any number of covariates. The procedure is in two steps: first, run an OLS regression

$$\begin{aligned} MPC_i &= (\beta^m)' \mathbf{Z_i} + \epsilon_i^m \\ URE_i &= (\beta^u)' \mathbf{Z_i} + \epsilon_i^u \end{aligned}$$

where $\mathbf{Z_i} = (1, Z_{i1}, \cdots, Z_{iJ})'$ is now a vector of covariates. Then, recover fitted values

$$\begin{aligned} \widehat{MPC_i} &= \left(\widehat{\beta^m}\right)' \mathbf{Z_i} \\ \widehat{URE_i} &= \left(\widehat{\beta^u}\right)' \mathbf{Z_i} \end{aligned}$$

and residuals $\widehat{\epsilon_i^m}$, $\widehat{\epsilon_i^u}$. The law of total covariance can now be expressed as

$$\text{Cov}\left(MPC_i, URE_i\right) = \text{Cov}\left(\widehat{MPC_i}, \widehat{URE_i}\right) + \text{Cov}\left(\widehat{\epsilon_i^m}, \widehat{\epsilon_i^u}\right) \tag{B.1}$$

The first term gives the component of explained covariance, and the second the component of unexplained covariance. The explained part of the covariance can be further decomposed as

$$\text{Cov}\left(\widehat{MPC_i}, \widehat{URE_i}\right) = \text{Cov}\left(\sum_{j=1}^{J} \widehat{\beta_j^m} Z_{ij}, \sum_{k=1}^{J} \widehat{\beta_k^m} Z_{ik}\right) = \sum_{j=1}^{J} \sum_{k=1}^{J} \widehat{\beta_j^m} \widehat{\beta_k^m} \text{Cov}\left(Z_{ij}, Z_{ik}\right) \tag{B.2}$$

Of course, the 'share of explained covariance' attributed to one particular covariate through this procedure depends on which other covariates are included in $\mathbf{Z_i}$.

Implementation. Tables B.10 reports the full matrix described by equation (B.2) for each of my three main covariances $\mathcal{E}_R$, $\mathcal{E}_P$, and $\mathcal{E}_Y$ in the SHIW, when all covariates from table 4 are included simultaneously. In the PSID and the CE, this exercise is less interesting since MPCs are only available at the group level, but it is possible to do by using the average value of explanatory variables in each bin. These results can easily be generated using the code provided online.

Table B.10: Fraction of $\mathcal{E}_R$ explained by each pair of SHIW covariates

	Age	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	9.03	0.20	-0.03	-2.36	0.13	-0.02	0.01	0.13
Male	0.83	1.78	0.13	0.25	-0.03	0.09	0.00	0.04
Married	-0.27	0.29	0.22	0.28	-0.09	-0.04	0.00	0.01
Years of ed.	-3.04	0.08	0.04	7.61	-0.05	0.49	-0.03	0.00
Family size	2.65	-0.15	-0.19	-0.82	0.38	0.34	-0.00	0.08
Res. South	-0.34	0.34	-0.06	6.09	0.27	10.53	-0.00	0.39
City size	0.59	0.08	0.04	-2.37	-0.01	-0.01	0.45	0.01
Unemployed	1.62	0.13	0.01	0.03	0.05	0.31	0.00	1.07

Table B.11: Fraction of $\mathcal{E}_P$ explained by each pair of SHIW covariates

	Age	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	13.29	0.22	-0.06	-2.56	1.52	-0.01	-0.02	-0.15
Male	1.22	1.98	0.28	0.28	-0.35	0.04	-0.01	-0.05
Married	-0.40	0.33	0.49	0.30	-1.06	-0.02	-0.02	-0.01
Years of ed.	-4.47	0.09	0.08	8.27	-0.60	0.22	0.12	-0.00
Family size	3.89	-0.16	-0.43	-0.89	4.48	0.15	0.01	-0.10
Res. South	-0.50	0.37	-0.13	6.62	3.12	4.84	0.00	-0.46
City size	0.87	0.08	0.10	-2.58	-0.14	-0.00	-1.73	-0.01
Unemployed	2.39	0.15	0.03	0.03	0.60	0.14	-0.00	-1.26

Table B.12: Fraction of $\mathcal{E}_Y$ explained by each pair of SHIW covariates

	Age	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	8.63	0.21	-0.14	-4.65	-2.33	-0.07	-0.04	0.48
Male	0.79	1.90	0.65	0.50	0.53	0.29	-0.02	0.17
Married	-0.26	0.31	1.12	0.55	1.63	-0.12	-0.03	0.04
Years of ed.	-2.90	0.08	0.19	15.00	0.93	1.62	0.19	0.01
Family size	2.53	-0.15	-0.98	-1.61	-6.89	1.13	0.02	0.31
Res. South	-0.32	0.36	-0.30	12.01	-4.79	35.21	0.01	1.49
City size	0.56	0.08	0.22	-4.68	0.22	-0.03	-2.66	0.04
Unemployed	1.55	0.14	0.07	0.05	-0.92	1.04	-0.01	4.09

C Details on the structural model of section 5

C.1 Additional details on the model

In the model, every household i has felicity function $u(c) = \frac{c^{1-\sigma^{-1}}}{1-\sigma^{-1}}$ and picks the sequence $\{c_t^i\}$ to maximize

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\left(\prod_{\tau=0}^{t}\beta_\tau^i\right)u\left(c_t^i\right)\right] \tag{C.1}$$

by choice of a portfolio of nominal bonds Λ_t and real bonds χ_t, with

$$P_t c_t^i + Q_t\left(\Lambda_{t+1}^i - \delta\Lambda_t^i\right) + q_t P_t\left(\chi_{t+1}^i - \delta\chi_t^i\right) = P_t y_t\left(e_t^i\right) + \Lambda_t^i + P_t\chi_t^i \tag{C.2}$$

and borrowing constraint

$$Q_t\Lambda_{t+1}^i + q_t P_t\chi_{t+1}^i \geq -\overline{D}_t P_t \tag{C.3}$$

Given Λ_t^i and χ_t^i, define the equivalent real bond position as

$$\lambda_t^i \equiv \frac{\Lambda_t^i}{P_{t-1}} + \frac{q_{t-1}}{Q_{t-1}}\chi_t^i$$

Along any perfect-foresight path with a constant price level $P_t = P$, no arbitrage between nominal and real bonds implies

$$\frac{1+\delta Q_t}{Q_{t-1}} = \frac{1+\delta q_t}{q_{t-1}}$$

and therefore $\frac{q_t}{Q_t} = \frac{1}{P}$. The consumer is then indifferent between holding nominal or real bonds. I resolve the indeterminacy by assuming that a constant share κ of the portfolio is invested in real (indexed) bonds, so that the household's portfolio allocation is

$$\begin{aligned}\frac{\Lambda_t^i}{P_{t-1}} &= (1-\kappa)\lambda_t \\ \frac{q_{t-1}}{Q_{t-1}}\chi_t^i &= \kappa\lambda_t\end{aligned}$$

With this notation, the budget constraint (A.55) and borrowing constraint (C.3) rewrite

$$\begin{aligned} c_t^i + Q_t\lambda_{t+1}^i &= y_t\left(e_t^i\right) + (1+\delta Q_t)\left[\frac{1-\kappa}{\Pi_t} + \kappa\right]\lambda_t \\ Q_t\lambda_{t+1}^i &\geq -\overline{D_t}\end{aligned} \tag{C.4}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is inflation. Along any perfect-foresight paths I consider, $\Pi_t = 1$ and the budget constraint simplifies to

$$c_t^i + Q_t\lambda_{t+1}^i = y_t\left(e_t^i\right) + (1+\delta Q_t)\lambda_t^i$$

Equation (C.4) to determine portfolio losses in case of a deviation of Π_t from its perfect-foresight value.

This problem has the following recursive formulation. The consumer's idiosyncratic state is given by the combination of s_t^i and his real bond position $\lambda_t^i \equiv \frac{\Lambda_t^i}{P_{t-1}}$. From his point of view, the relevant components of the aggregate state are $\left(y_t, Q_t, \Pi_t, \overline{D}_t\right)$, where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the inflation rate at t. Hence his optimization problem is characterized by the Bellman equation:

$$\begin{aligned} V_t(\lambda, \mathbf{s}) = \quad & \max_{c,\lambda'} \quad u(c) + \beta(\mathbf{s})\, \mathbb{E}\left[V_{t+1}(\lambda', \mathbf{s}')\,|\mathbf{s}\right] \\ & \text{s.t.} \quad c + Q_t\lambda' = y(\mathbf{s}) + (1+\delta Q_t)\lambda \\ & \qquad\quad Q_t\lambda' \geq -\overline{D}_t \end{aligned} \tag{C.5}$$

I calibrate the model such that, at the initial steady-state distribution $\Psi(\mathrm{s}, \lambda)$, aggregate consumption is equal to the aggregate endowment, so

$$C_t \equiv \int c_t(\mathbf{s}, \lambda)\, d\Psi_t(\mathbf{s}, \lambda) = Y_t$$

This can be interpreted as the flexible-price equilibrium of a Huggett model with no government debt.

C.2 Behavior of constrained agents after real interest rate shocks

I specify that the borrowing limit $\{\overline{D}_t\}$ adjusts in response to such a shock so as to hold the *real coupon payment in the next period fixed*: $\overline{D}_t = Q_t\overline{d}$, or equivalently

$$\frac{\Lambda_{t+1}^i}{P_t} \geq -\overline{d} \tag{C.6}$$

In addition to being a natural one, the specification of the adjustment process for borrowing limits in (C.6) implies that theorem 1 holds exactly, including for agents at a binding borrowing limit. It is crucial to understand how these agents are affected depending on the maturity of the debt in the economy, δ. In the experiments I consider inflation is $\Pi_t = 1$, so that nominal and real interest rates are equal. Consider an agent with income Y_t^i who maintains himself at the borrowing limit in an initial steady-state where the real interest rate is R and the bond price is constant at $Q = \frac{1}{R-\delta}$. His consumption is equal to his income, minus the interest payment on the value of the borrowing limit $\overline{D} = Q\overline{d}$:

$$c_t^i = Y_t^i - (R-1)\,\overline{D}$$

Across economies with different debt maturities δ, $\overline{D}$ is a constant, so that the steady-state payments are the same, but the exposure of these payments to real interest rate changes differ. Indeed we can decompose:

$$(R-1)\,\overline{D} = (R-\delta)\,\overline{D} - \overline{D}\,(1-\delta) = \overline{d} + \underline{URE}$$

where $\overline{d} \equiv (R-\delta)\,\overline{D}$ is the part that is precontracted and $\underline{URE} \equiv -\overline{D}\,(1-\delta)$ the part that is subject to interest changes. Hence, economies with different δ involve very different levels of unhedged interest rate exposures for borrowing-constrained agents, ranging from the full principal $-\overline{D}$ when $\delta = 0$ to none when $\delta = 1$. In the benchmark calibration with $\delta = 0.95$, highly-indebted low-income agents use their full income for interest payments and amortization $\overline{d}$, and then borrow, as on a home equity line of credit, to maintain their consumption level. Hence they are only mildly affected by changes in interest rates. On the other hand, when all debt is short-term, we instead have $c_t^i = Y_t^i - (R_t - 1)\,\overline{D}$ for constrained agents, leading to large swings in their consumption as interest rates change.

C.3 Computational method

Method of endogenous gridpoints

I use the method of endogenous gridpoints (Carroll 2006) to solve for consumer policy functions. This is a computationally efficient solution method based on policy function iteration, which avoids costly root-solving operations and is applicable to any standard incomplete market problem with CRRA utility functions (see for example Guerrieri and Lorenzoni 2015). The computation involves finding the policy function for consumption $c_t\,(\lambda, \mathrm{s})$ on a fine grid for λ (2000 points) and a discrete grid for s (20 points: 2 states for β and 10 states for z).

When the borrowing constraint binds, which happens for $\lambda \leq \lambda_t^*$ for some λ_t^*, the policy function is given by

$$c_t\,(\lambda, \mathrm{s}) = y_t\,(\mathrm{s}) + \lambda\,(1 + Q_t\delta) + \overline{D_t} \tag{C.7}$$

For $\lambda > \overline{\lambda_t}$ the borrowing constraint is not binding, and defining the real interest rate by

$$R_t = \frac{1 + \delta Q_{t+1}}{Q_t}$$

the solution is characterized by the Euler equation

$$c_t^{-\sigma^{-1}} = \beta_t R_t \mathbb{E}_t \left[(c_{t+1})^{-\sigma^{-1}} \right] \tag{C.8}$$

The idea behind endogenous gridpoints is to start from a given state today s and a *target* bond level in the next period λ'. The budget constraint

$$\lambda' = \frac{1}{Q_t} \left(y_t \left(\mathrm{s} \right) + \lambda \left(1 + Q\delta \right) - c \right) \tag{C.9}$$

implies that the pairs (λ, c) that are consistent with λ' are on a straight line. Moreover, given a guess for the policy function $c_{t+1}(\cdot, \cdot)$, there is a unique value of g consistent with an optimal choice of λ' tomorrow, given by

$$c = \left(\beta_t R_t \mathbb{E} \left[c_{t+1} \left(\lambda', \mathrm{s}' \right)^{-\sigma^{-1}} | \mathrm{s} \right] \right)^{-\sigma} \tag{C.10}$$

Hence by varying the target bond level λ', one traces out the policy function $c_t(\lambda, \mathrm{s})$ in the region $\lambda > \overline{\lambda_t}$. This is very efficient computationally since it can be performed on the grid for λ' which is used to store $c_{t+1}(\lambda', \mathrm{s}')$. The calculation only involves:

a) Finding c using (C.10), which only involves power operations and linear combinations using the Markov transition matrix for s

b) Finding λ by solving one linear equation in one unknown in (C.9)

c) Defining λ_t^* as the bond value today that corresponds to $\lambda' = \frac{\overline{D}_t}{Q_t}$, since this is the highest level of bonds for which the consumer chooses to be at the borrowing limit tomorrow with his Euler equation holding with equality

d) If $\lambda_t^* > \frac{\overline{D}_{t-1}}{Q_{t-1}}$, completing the policy function on an arbitrary grid for $\left[\frac{\overline{D}_{t-1}}{Q_{t-1}}, \lambda_t^* \right]$ using (C.7)

e) Interpolating the resulting policy function back to the grid for λ

Figure C.1 illustrates the construction of the policy function for state $\mathrm{s} = 1$ in the model calibration. Consider targeting a bond level $\lambda' = 0$. This yields a value for consumption through the Euler Equation (C.8) indicated by the dashed yellow line. It also yields a set of pairs (c, λ) consistent with $\lambda' = 0$ through the budget constraint (C.9), as indicated by the solid purple line. The intersection of these two lines yields a new point of the policy function over λ. Varying λ' in this way we trace out this policy function (solid blue line) over the range where the Euler equation holds. The policy function is completed by the set of points consistent with borrowing at the limit (solid red line).

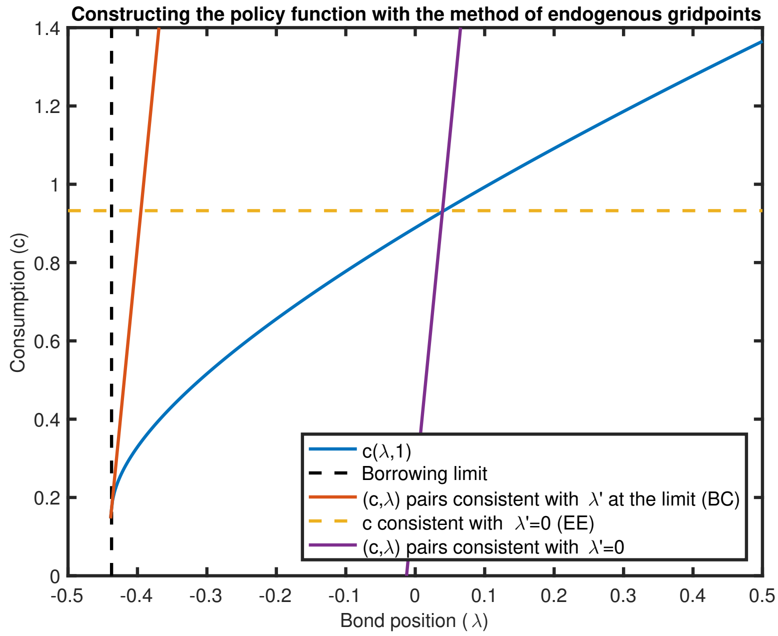

Figure C.1: Constructing the policy function $c\left(\lambda, \mathrm{s}=1\right)$ **(model calibration)**

Flexible price steady-state

In a steady-state, the consumer faces a constant sequence $\left(Q_t, \Pi_t, \overline{D}_t\right) = \left(\frac{1}{R-\delta}, 1, \overline{D}\right)$ where R is the interest rate that prevails in steady-state.

I find the steady-state interest rate R using the following classic bisection procedure:

a) Start with a guess for R and for the consumption policy function $c^0\left(\lambda, \mathrm{s}\right)$

b) Iterate on $c_t\left(\lambda, \mathrm{s}\right)$ using the procedure described in C.3 until $c_{t+1} - c_t$ is sufficiently small. By construction, $c = c^{SS}(b, y)$ then satisfies the functional equation

$$c\left(\lambda, \mathrm{s}\right)^{-\sigma^{-1}} = \beta\left(\mathrm{s}\right) R\mathbb{E}\left[c\left(\frac{1}{Q}\left(y\left(\mathrm{s}\right) + \lambda\left(1+\delta\right) - c\left(\lambda, \mathrm{s}\right)\right), \mathrm{s}'\right)^{-\sigma^{-1}} | \mathrm{s}\right] \quad \text{(C.11)}$$

c) Use the inverse policy function for next period bonds $\lambda\left(\lambda', \mathrm{s}\right) = \left[\lambda'\right]^{-1}\left(\lambda', \mathrm{s}\right)$, which is computed as part of the endogenous gridpoints method, to find the stationary conditional distribution for bonds $\Psi\left(\lambda|\mathrm{s}\right)$, as the fixed point of the operator mapping Ψ_t to Ψ_{t+1},

$$\Psi_{t+1}\left(\lambda'|\mathrm{s}'\right) = \sum_{\mathrm{s}} \Psi_t\left(\left[\lambda'\right]^{-1}\left(\lambda', \mathrm{s}\right)|\mathrm{s}\right) \frac{\Pr\left(\mathrm{s}_t = \mathrm{s}\right)}{\Pr\left(\mathrm{s}_{t+1} = \mathrm{s}'\right)} \Pi\left(\mathrm{s}'|\mathrm{s}\right)$$

d) Check that goods market clear, $\int c(\lambda; \mathrm{s})\, d\Psi(\lambda, \mathrm{s}) = Y^*$. If they do not, adjust R in the direction of market clearing and repeat (one must first determine whether steady-state consumption is locally increasing or decreasing in R)

Transitional dynamics following a shock

Here I describe how to compute perfect-foresight transition paths following a change in the path for real interest rates $\{R_t\}$, such as at described by equation (29). Assume that the economy returns to steady-state by time T (in my computations, $T = 200$ when the shock has persistence $\rho = 0.5$)

a) Using the path of R_t, compute the bond price path

$$Q_t = \frac{1 + \delta Q_{t+1}}{R_t}$$

backwards starting from $Q_T = \frac{1}{R-\delta}$.

b) Given the paths for $\left(Q_t, \Pi_t = 1, \overline{D}_t\right)$, compute policy functions backwards, starting from $c_T = c^{SS}$, using the method of endogenous gridpoints described above.

c) Starting from the conditional bond distribution that prevails in the initial steady-state, and using the transitional inverse policy function for next period bonds computed as part of step b), compute the conditional bond distributions along the transition using

$$\Psi_{t+1}(\lambda'|\mathrm{s}') = \sum_{\mathrm{s}} \Psi_t\left(\left[\lambda'_t\right]^{-1}(\lambda', \mathrm{s})\,|\mathrm{s}\right) \frac{\Pr(\mathrm{s}_t = \mathrm{s})}{\Pr(\mathrm{s}_{t+1} = \mathrm{s}')} \Pi(\mathrm{s}'|\mathrm{s})$$

d) Finally, compute aggregate consumption as $C_t = \int c_t(\lambda; \mathrm{s})\, d\Psi_t(\lambda, \mathrm{s})$.